4OR manuscript No.
(will be inserted by the editor)

Non-classical preference models in combinatorial problems: models and algorithms for graphs

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Received: date / Revised version: date

Abstract This is a summary of the most important results presented in the author's PhD thesis (Spanjaard, 2003). This thesis, written in French, was defended on 16 December 2003 and supervised by Patrice Perny. A copy is available from the author upon request. This thesis deals with the search for preferred solutions in combinatorial optimization problems (and more particularly graph problems). It aims at conciliating preference modelling and algorithmic concerns for decision aiding.

Keywords: Decision Aiding, Combinatorial Optimization, Preference Modelling, Graphs, Heuristic Search

1 Introduction

Comparisons of solutions in most combinatorial problems is often based on an additive cost function inducing a complete order on solutions. The computational efficiency of classical algorithms strongly relies on this assumption. This thesis investigates these problems when costs are not reducible to such a function.

2 Combinatorial decision problems

The *classical model of combinatorial optimization* makes use of an additively decomposable objective function. However, many real-world problems are not reducible to such a model:

- In problems of equitable allocation of earth observing satellites resources, a vector of individual utilities (with respect to the differents agents) is associated to each solution. Lemaître et al. (2003) suggest to resort to the leximin criterion (e.g., Dubois et al., 1996) to compare solutions. This criterion consists in reordering the vector components in increasing order, and then comparing lexicographically the obtained vectors.
- When associating a vector (rather than a scalar value) to a solution, a classical comparison rule between vectors is the Pareto-dominance relation. A feasible solution is called non-dominated, according to Pareto-dominance, if there is no other feasible option that surpasses this solution on any of the criteria without reducing performance on another criterion.
- Finally, consider a communication network where each arc represents a link between two vertices, and assume that there is an upper bound on the quantity of information that can travel through each link. A natural problem is then to find a path of maximum capacity, i.e., such that the minimum flow along the links is the greatest as possible.

All these problems do not reduce to the classical model of combinatorial optimization. We call them *combinatorial decision problems*, in which both the set of solutions and the preference are not defined explicitly but implicitly, and preferences are not reducible to an additively decomposable objective function.

3 Preferred trees and paths

We have investigated combinatorial problems in which preferences take the form of a quasi-transitive binary relation (i.e., the asymmetric part of which is transitive) defined on the solutions space. We have proposed preference-based search algorithms for two classical problems, namely:

- the preferred spanning trees problem (a generalization of the minimum spanning tree problem),
- the preferred paths problem (a generalization of the shortest path problem).

We have proposed preference-based counterparts of standard algorithms used for the minimal spanning tree problem and the shortest path problem. We have identified a very useful axiom for preference relations, that we have called *preadditivity* (namely, when the comparison of two solutions does not depend on their common part). Using this axiom, we have established admissibility results concerning our preference-based search algorithms. Furthermore, we have adressed the problem of dealing with non-preadditive preference relations and we have provided different possible solutions for different particular problems : lower approximation of the set of preferred solutions for multicriteria spanning trees problems, relaxation of the preadditivity axiom for interval-valued preferred path problems. The detailed results can be found in Perny and Spanjaard (2004).

4 Preference-based search in state-space graphs

Many practical problems considered in Artificial Intelligence (e.g., path planning, game search, web search) can be modelled as the search of an optimal path from an initial node to a goal node in a state space graph defined implicitly. In this framework, useful constructive search algorithms have been proposed, performing implicit enumeration of feasible solutions, directed by a numerical cost function to be minimized. Among them, the most famous algorithm is called A* (Hart et al., 1968), and it guarantees to find the best solution without generating the whole state space graph (to a certain extent). We have presented PBA* algorithm (Preference-Based A*), a generalization of the A* algorithm, designed to process quasi-transitive preference relations defined over the set of solutions. Then, considering a particular subclass of preference structures characterized by the two axioms of preadditivity and *monotonicity* (one prefer a subset of a set to that set), we have established termination (the algorithm stops after a finite number of iterations), completeness (the algorithms yields to at least one solution as soon as a solution exists) and admissibility (the algorithm guarantees to terminate with the whole set of preferred solutions) results for PBA*. The detailed results are presented in Perny and Spanjaard (2002).

5 Bottleneck shortest paths on a partially ordered scale

In bottleneck combinatorial problems, admissible solutions are compared with respect to their maximal elements. In such problems, one may work with an ordinal evaluation scale instead of a numerical scale. We have considered a generalization of this problem in which one only has a partially ordered scale (instead of a completely ordered scale). We have introduced a mappimax comparison operator between sets of evaluations (which reduces to the classical max operator when the order is complete) and we have established computational complexity results for this variation of the shortest path problem. Finally, we have formulated our problem as an algebraic shortest path problem (e.g., Rote, 1990; Gondran and Minoux, 2001) and we have suggested adequate algorithms to solve it in the subsequent semiring (an algebraic structure that allows to unify the solving methods in many path problems). The detailed results are presented in Monnot and Spanjaard (2003).

6 Robust solutions in state space graphs with multiple scenarios

Recent works aim at handling uncertainty in heuristic search. In this concern, many papers focus on problems where the cost of the arcs are ill-known and characterized by uncertainty distributions (Wellman et al., 1995; Wurman and Wellman, 1996). We have studied another variation of the search problem under uncertainty, that concerns situations where costs of paths might depend on different possible scenarios (states of the world) or come from discordant sources of information. Our aim was to focus on the idea of robustness. We have presented axiomatic requirements for preference compatibility with the intuitive idea of robustness. This has led us to propose the Lorenz dominance rule as a basis for robustness analysis. We have presented complexity results about the determination of robust solutions and we have proposed algorithmic solutions. Finally, we have established an axiomatic justification of the refinement of robustness by an OWA (Ordered Weighted Average) criterion. The detailed results are presented in Perny and Spanjaard (2003).

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