Du vote à la théorie de l'argumentation

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séminaire LIP6, 27 Juin 2011

Basé sur les travaux suivants:

- *Possible winners when new candidates are added*. (avec Y. Chevaleyre, J. Lang, J. Monnot). AAAI-2010.
- *Compilation and Communication protocols for voting with a dynamic set of candidates* (avec Y. Chevaleyre, J. Lang, J. Monnot). TARK-2011.
- *On the Outcomes of Multiparty Persuasion*. (avec E. Bonzon). AAMAS-2011

1. INTRODUCTION

2. VOTE EN CONTEXTE DYNAMIQUE
   - Possible winners determination
   - Elicitation protocols
   - Related Works

3. ARGUMENTATION DE GROUPE
   - Argumentation basics
   - A multiparty protocol
   - Properties

Motivation: Vote en ligne

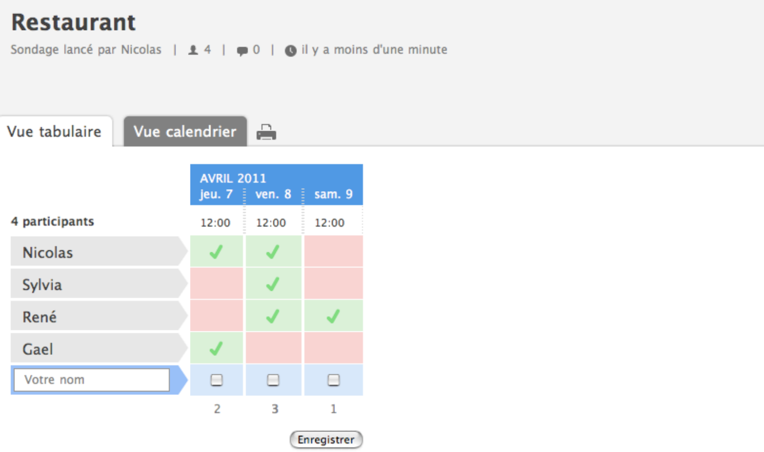
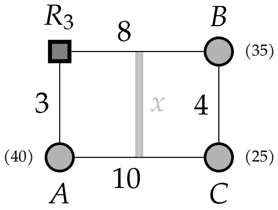


Figure: Dimanche le resto est peut-être ouvert, à vérifier

MOTIVATION: COORDINATION MULTI-AGENTS



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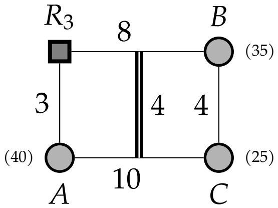


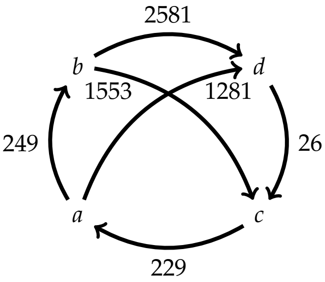
VOTE

$\mathcal{N} = \{1, \ldots, n\}$ les votants, et $X = \{a, b, \ldots\}$ les p candidats

- un vote un ordre linéaire sur l'ensemble des candidats
- un profil P est la collection des votes des votants
- une règle de vote r associe à un profil un (unique) vainqueur $r(P)$
- différentes règles de vote:
  - règles de scoring, où un vecteur de scores associe un nombre de points à chaqu position dans le vecteur
  - règles baées sur comparaison par paires des candidats
  - règles procédant par élimination successives de candidats

An Example

Take the following profile P involving 4703 voters and the corresponding (weighted) majority graph $\mathcal{M}_P$



1340:	$a \succ b \succ c \succ d$
827:	$b \succ a \succ d \succ c$
75:	$d \succ a \succ b \succ c$
961:	$c \succ a \succ b \succ d$
1400:	$d \succ b \succ c \succ a$
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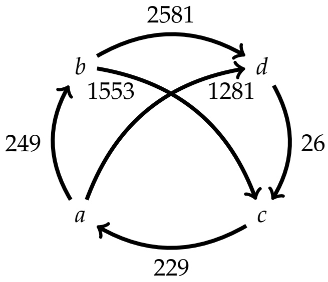

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Plurality winner: d

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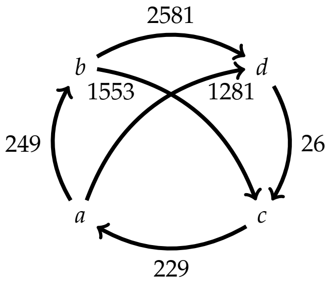

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Borda winner: a

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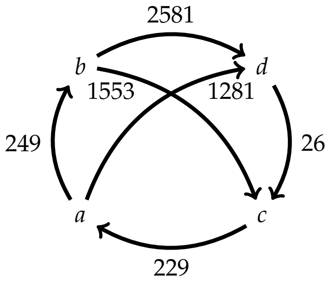

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Simpson winner: a

ENSEMBLE DYNAMIQUE DE CANDIDATS

On distingue :

- X l'ensemble des candidats déclarés lors du vote, et
- Y l'ensemble des nouveaux candidats.

Soit P_X le profil du votes déjà effectué sur les candidats de X. Sachant que les candidats de Y peuvent se déclarer, au moins deux approches sont possibles:

1. faire avec: peut-on déterminer les candidats qui sont gagnants possibles étant donné l'arrivée de nouveaux candidats?
2. éliciter: quelle est la meilleure manière d'éliciter les votes concernant les nouveaux candidats Y?

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Possible Winners

- Given a voting situation π and a voting rules r, $x \in X$ is a possible winner (wrt π and r) if there is a profile P extending P_X st. $r(P) = x$

Note that the necessary winner problem is not very relevant here, any new candidate being (under mild conditions) a possible winner.

Question

Study the possible winner problem with new candidates focusing on scoring rules $\langle s_1, \ldots, s_p \rangle$ ($s_i \geq s_{i+1}$ and $s_1 > s_p$).

EXAMPLE

Example (Plurality, 1 new candidate)

1: $a \succ d \succ c \succ b$
2: $a \succ b \succ c \succ d$
3: $a \succ d \succ c \succ b$
4: $d \succ a \succ c \succ b$
5: $b \succ a \succ c \succ d$
6: $b \succ d \succ a \succ c$
7: $c \succ d \succ a \succ b$
8: $c \succ b \succ d \succ a$

Tie-breaking: $a > b > c > d > y$

Plurality scores:
$s(a) = 3, s(b) = 2, s(c) = 2, s(d) = 1$

Who are the possible winners?
certainly a is...

Example

Example (Plurality, 1 new candidate)

1: $a \succ d \succ c \succ b \succ y$
2: $y \succ a \succ b \succ c \succ d$
3: $y \succ a \succ d \succ c \succ b$
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b is as well...

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Who are the possible winners?
c is not.

PLURALITY: AN EASY CASE

The general condition is easy to state. Intuitively:

- each new candidate can be placed at the top to decrease the score of a candidate;
- for each candidate with a higher score than x we must put the new candidate on top a number of times equal to the difference of scores (+1 if that candidate has priority in the tie-breaking rule);
- the score of the new candidate must not be higher (or indeed equal if the new candidate has priority) than the current score of x.

Generalizes to k new candidates.

$$top(P_X, x) \geq \frac{1}{k} \sum_{z \in X} max(0, top(P_X, z) - top(P_X, x))$$

MORE RESULTS...

Not always easy (even for scoring rules)...

- easy for K-approval if $K \leq 2$ or $k \leq 2$, then becomes hard.
- easy for Borda ($\langle 5, 4, 3, 2, 1, 0 \rangle$)
- may be hard even for a single new candidate ($\langle 3, 2, 1, 0, 0, 0 \rangle$)

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Protocoles de mémorisation / communication

On parle de MC-protocole, associant:

- une fonction de mémorisation σ
- un protocole d'élicitation π

On peut alors évaluer un MC-protocole de manière bi-critère:

- l'espace requis (nbre de bits) par la fonction de mémorisation σ
- la quantité de communication requise (nbre de bits) par le protocole π

Attention: le coût des bits de stockage et de communication n'est pas nécessairement le même (cf. vote en ligne vs. coordination multi-agents).

MÉMORISATION DES PROFILS

Comment mémoriser le profil?

1	2	3
a	b	b
b	c	a
c	a	c

↓

Elicitation
des informations
manquantes

↓

Gagnant: x

Mémorisation des Profils

Comment mémoriser le profil?

1	2	3
a	b	b
b	c	a
c	a	c

→ *fonction de mémorisation* → 10011000101

↓

Elicitation des informations manquantes

↓

Gagnant: x ↔ *même gagnant* ↔ Gagnant: x

FONCTIONS DE MÉMORISATION

Prenons l'exemple de la mémorisation anonyme:

Profil:

1	2	3	4	5
a	b	b	c	a
b	c	a	a	b
c	a	c	b	c

Mémorisation anonyme:

2	1	1	1
a	b	b	c
b	c	a	a
c	a	c	b

Pour un grand nombre de votants, cette mémorisation est bien moins coûteuse.

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Pour un grand nombre de votants, cette mémorisation est bien moins coûteuse.

- $n \log p!$ pour la mémorisation complète, vs. $p! \log n$ pour la mémorisation anonyme.
  Pour $n = 4703$ et $p = 4$ on a 312 ($24 \log 4703$) vs. 23515 bits ($4703 \log 24$).
- fonctions de mémorisation spécifiques à certaines règles

Illustration: le cas de la K-approbation

On donne 1 point à chaque candidat classé dans les K premiers

1	2	3	4	5
a	b	b	c	a
b	c	a	a	b
c	a	c	e	c
d	e	d	d	d
e	d	e	b	e

scores: $\langle a:5, b:4, c:5, d:0, e:1\rangle$

- quel protocole en cas de mémorisation complète?
  - ce protocole est-il optimal pour cette compilation?
  - ce CC-protocole est-il Pareto-optimal?
- quel protocole en cas de mémorisation anonyme?

MÉMORISATION COMPLÈTE

P_X est mémorisé intégralement. Il suffit de déterminer quels sont les candidats approuvés parmi ceux de Y.
Deux (sous-)protocoles possibles:

(1) demander à chaque votant la liste (dans un ordre donné) des k nouveaux candidats marqués approuvé ou non
$\Rightarrow n \cdot k$ bits

(2) demander la liste avec les identités des candidats approuvés
$\Rightarrow n \cdot K \log k$

Note: utiliser (1) si $k > K \log k$, et (2) sinon.

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Note: utiliser (1) si $k > K \log k$, et (2) sinon.
Prenons $K = 3$, et avec $k = 5$ nouveaux candidats.
(1) demande 5 bits tandis que (2) demande 9 bits.

Mémorisation Complète

Le protocole de communication proposé est-il optimal pour la mémorisation complète?

- comment montrer qu'aucun autre protocole ne pourrait être meilleur que celui proposé?

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  Prenons $K = 3$, et avec $k = 5$ nouveaux candidats.
  (1) demande 5 bits tandis que (2) demande 6 bits.
- pour répondre à cette question, techniques de complexité de communication (*fooling sets*) montrant que le problème requiert d'échanger un certain nombre de bits.
- Résultat: Le protocole π est (asymptotiquement) optimal pour la compilation complète.

MÉMORISATION SEMI-ANONYME

Pourtant le CC-protocole $\langle \pi, \sigma_F \rangle$ n'est pas Pareto-optimal...
On peut le montrer en exhibant une mémorisation plus frugale mais suffisante pour appliquer le protocole π.

- il suffit de mémoriser la liste des K premiers candidats pour chaque votants.

Mémorisation semi-anonyme

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- il suffit de mémoriser la liste des K premiers candidats pour chaque votants.
- si $k \leq K$, on peut même se contenter de mémoriser sous forme de liste les candidats en position" éjectable" :

1	2	3	4	5	
a	b	b	c	a	score garanti:$\langle a:2, b:2, c:1, d:0, e:0 \rangle$
b	c	a	a	b	positions éjectables:$\langle \langle b,c \rangle, \langle c,c \rangle, \langle a,c \rangle, \langle a,e \rangle, \langle b,c \rangle \rangle$
c	a	c	e	c	
d	e	d	d	d	
e	d	e	b	e	

MÉMORISATION ANONYME

On oublie les identités des votants.

- on ordonne les candidats tq. $score(x_1) \geq score(x_2) \geq \ldots$
- pour chaque votant i demander:
  "approuvez-vous au moins un candidat de Y?"
  - si "oui" alors
    - éliciter lesquels ($voteY(i)$)
    - mettre à jour les scores de Y
    - $posEjectable(i) \leftarrow [K : K - |voteY(i)| + 1]$
- $y^* \leftarrow \text{argmax}_y score(y)$
- si $score(y^*) > score(x_1)$ alors retourner y^*

  sinon
  - $X_{pot} \leftarrow$ gagnants "potentiels" de X
  - pour chaque votant i tq. $|voteY(i)| > 0$ demander:
    "qui parmi X_{pot} était en $posEjectable(i)$?"

MÉMORISATION ANONYME: EXEMPLE

Exemple: $K = 2, k = 2, n = 5$

(1) $\langle a : 2, b : 2, c : 2, d : 2, e : 2 \rangle$

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(1) $\langle a : 2, b : 2, c : 2, d : 2, e : 2 \rangle$

(2) 1 vote $\{y_1, y_2\}$, 2 vote $\{y_1\}$, 3, 4 et 5 votent $\emptyset$

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$\langle y_1 : 2, y_1 : 1 \rangle$

(3) $\langle a : 2, b : 2, c : 2, d : 2, \cancel{e : 2} \rangle$

- qui parmi $\{a, b, c, d\}$ était en position éjectable?
- 1 dit $\{a, b\}$, 2 dit $\{d\}$

MÉMORISATION ANONYME: EXEMPLE

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(3) $\langle a : 2, b : 2, c : 2, d : 2, \not{e}\,\not{:}\,\not{2} \rangle$

- qui parmi $\{a, b, c, d\}$ était en position éjectable?
- 1 dit $\{a, b\}$, 2 dit $\{d\}$
- $\langle a : 1, b : 1, c : 2, d : 1, e : 2, y_1 : 2, y_2 : 1 \rangle$

$\Rightarrow$ c est le gagnant.

Mémorisation Anonyme: Exemple

Exemple: $K = 2, k = 2, n = 5$

(1) $\langle a : 2, b : 2, c : 2, d : 2, e : 2 \rangle$

(2) 1 vote $\{y_1, y_2\}$, 2 vote $\{y_1\}$, 3, 4 et 5 votent $\emptyset$
$\langle y_1 : 2, y_1 : 1 \rangle$

(3) $\langle a : 2, b : 2, c : 2, d : 2, \cancel{e : 2} \rangle$

- qui parmi $\{a, b, c, d\}$ était en position éjectable?
- 1 dit $\{a, b\}$, 2 dit $\{d\}$
- $\langle a : 1, b : 1, c : 2, d : 1, e : 2, y_1 : 2, y_2 : 1 \rangle$

$\Rightarrow$ c est le gagnant.

Résultat

ce protocole est aussi frugal pour les agents que π (mémoire complète)!

$\Rightarrow$ Intuition: si le nombre de questions à poser est grand, le nombre de gagnants potentiels est faible (et vice-versa).

Computational Social Choice

This work belong to the rapidly expanding field of *computational social choice*. Many other topics studied:

- complexity of computing the result of the election
  P. Faliszewski, E. Hemaspaandra, L. A. Hemaspaandra, J. Rothe. A Richer Understanding of the Complexity of Election Systems. CoRR, 2006.
- complexity of manipulating the election (by the candidates, by the chair): hardness may actually be a good thing here! But worst-case results are challenged...
  AI's War on Manipulation: Are We Winning?. P. Faliszewski and A. D. Procaccia. AI Magazine 31(4), 2010.
- communication complexity of voting rules
  V. Conitzer and T. Sandholm. Communication Complexity of Common Voting Rules. In Proceedings of EC-05
- and much more...
  Y. Chevaleyre, U. Endriss, J. Lang, and N. Maudet. A Short Introduction to Computational Social Choice. In Proc. SOFSEM 2007

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MOTIVATION: ONLINE DEBATE PLATFORMS

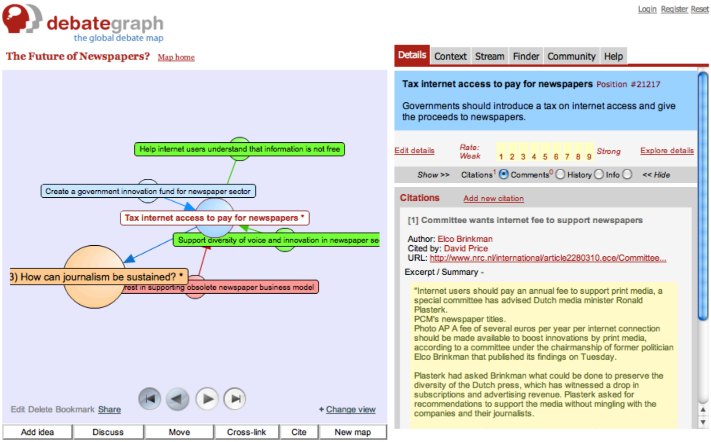


Figure: debategraph.org

Motivation: Online Debate Platforms

Some practical problems with these systems:

- the number of agents and arguments put forward give rise to unfocused debates, difficult to follow and interpret
- agents may have unequal access to the debate platform

Hence we need to regulate the debate.

A Simple Example

Consider the following arguments:

(c) The US army is preparing a secret plan to retreat from Afghanistan (source: Wikileaks)

(b) Our (informed) sources say the documents are fake. (source: NYT)

(a) The US media cannot be trusted on military issues (source: N. Chomsky)

ASSUMPTIONS

The following is taken for granted:

- *many* agents debating
- a *single* issue is under discussion
- *no coordination* takes place among agents
- agents *agree* on the set of arguments
- agents *may disagree* on the attack relations among these arguments

Plan for the rest of the talk

1. Basics of argumentation theory
2. A simple protocol for multiparty persuasion
3. Some properties

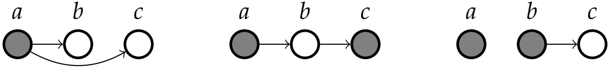
Argumentation systems

Abstract view of argumentation (no specification of the actual content of arguments), due to [Dung, 95]

- Argumentation system AS defined as a pair
  - A : set of arguments
  - R : attack relation ($\subseteq A \times A$)
- Argumentation graph

$$AS = \{a, b, c\}, \{(b, c), (a, b), (a, c)\}$$

WHY SYSTEMS MAY DIFFER



- a_1 thinks CHK is the more credible source, and sees WKL as a media (more credible than the NYT).
  a_1 - $\mathcal{E}(AS_1) = \{a\}$
- a_2 thinks NYT is more credible than WKL, but that CHK is more credible than NYT. But he believes WKL cannot be seen as a media.
  a_2 - $\mathcal{E}(AS_2) = \{a, c\}$
- a_3 thinks the NYT is the more credible source, and that CHK always say rubbish.
  a_3 - $\mathcal{E}(AS_3) = \{a, b\}$

ACCEPTABILITY

Now we need to define what (sets of) arguments should be considered as "justified" point of view.

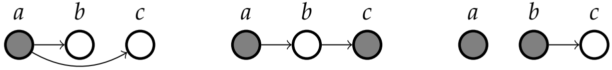
- *S collectively defends* $a \in A$ iff $\forall b \in A$ such that bRa, $\exists c \in S$ such that cRb
- S is a *grounded extension* iff S is the least fixed point of the characteristic function of AS
- F: $2^A \to 2^A$ with $F(S) = \{a$ such that S collectively defends $a\}$
- always exists a unique grounded extension, denoted by $\mathcal{E}(\mathsf{AS})$

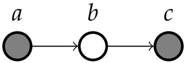
MERGED ARGUMENTATION SYSTEM

Introduced by [Coste-Marquis *et al.*, AIJ'07]

- n agents holding an argumentation system AS_i
- Majority Argumentation System
  - Attacks supported by a majority of agents
  - Ties broken in favour of the absence of an attack
- $MAS_N = \langle A, M \rangle$ where
  - $M \subseteq A \times A$
  - xMy when $|\{i \in N | xR_iy\}| > |\{i \in N | xR_iy\}|$
- Merged outcome: $\mathcal{E}(MAS_N)$

MERGED ARGUMENTATION SYSTEM: EXAMPLE





$$\mathcal{E}(MAS) = \{a, c\}$$

BASIC IDEAS OF THE PROTOCOL

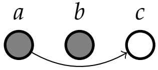
- agents prefer state of the debate where the issue has the same acceptability status as in their individual system.
- Convenient to distinguish two sets of agents:
  - $CON = \{a_i \in N | d \notin \mathcal{E}(\mathsf{AS}_i)\}$
  - $PRO = \{a_i \in N | d \in \mathcal{E}(\mathsf{AS}_i)\}$
- the proposed protocol is based on the principle of *direct relevance* [Prakken, 2001]: a move is valid iff it changes the current status of the issue.

A relevance-based multiparty protocol

- Agents report their individual view on the issue to the central authority, which then assign (privately) each agent to PRO or CON.
- The first round starts with the issue on the gameboard and the turn given to CON.
- Until a group of agents cannot move :
  - agents independently propose moves to the central authority;
  - the central authority picks the first (or at random) relevant move from the group of agents whose turn is active, update the gameboard, and passes the turn to the other group

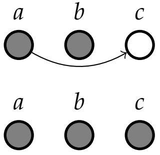
A multiparty protocol

$t = 1$ - a_1 plays for CON: $RP_1^1 = \{(a, c)\}$



$t = 1$ - a_1 plays for CON: $RP_1^1 = \{(a, c)\}$

$t = 2$ - a_2: $RP_2^2 = \{(a, c)\}$

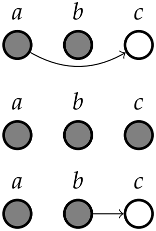


A multiparty protocol

$t = 1$ - a_1 plays for CON: $RP_1^1 = \{(a, c)\}$

$t = 2$ - a_2: $RP_2^2 = \{(a, c)\}$

$t = 3$ - a_3 plays for CON: $RP_3^3 = \{(b, c)\}$



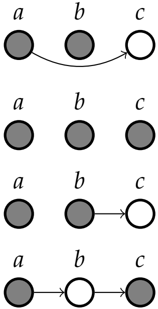
A multiparty protocol

$t = 1$ - a_1 plays for CON: $RP_1^1 = \{(a, c)\}$

$t = 2$ - a_2: $RP_2^2 = \{(a, c)\}$

$t = 3$ - a_3 plays for CON: $RP_3^3 = \{(b, c)\}$

$t = 4$ - a_2: $RP_2^4 = \{(a, b), (a, c)\}$



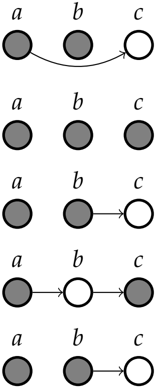
$t = 1$ - a_1 plays for CON: $RP_1^1 = \{(a, c)\}$

$t = 2$ - a_2: $RP_2^2 = \{(a, c)\}$

$t = 3$ - a_3 plays for CON: $RP_3^3 = \{(b, c)\}$

$t = 4$ - a_2: $RP_2^4 = \{(a, b), (a, c)\}$

$t = 5$ - a_3: $RP_3^5 = \{(b, c), (a, b)\}$
$t = 6$ - a_2 cannot add c in the extension



What are the outcomes with such a protocol?

The issue of the debate is a **possible outcome** for a group X if this group has a possibility to set the acceptability status of this argument coincide in the debate and in their individual system. The issue is a **necessary outcome** for X if this issue is not a possible outcome for $\overline{X}$.

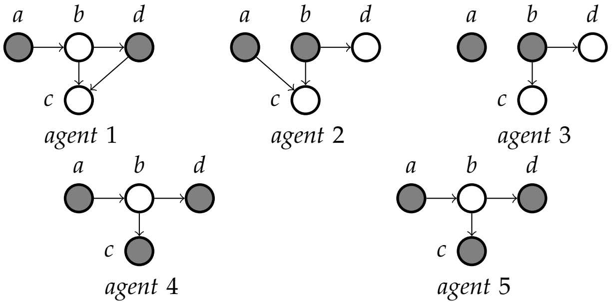
- is the outcome pre-determined from the initial situation?
- does the outcome coincide with the merged outcome?
- is it useful to allow reinforcement moves?

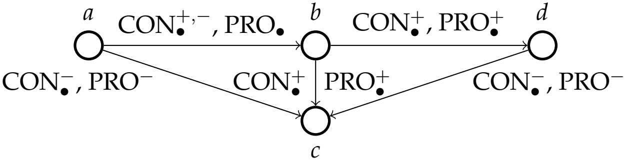
CONTROL OF AN EDGE OF THE GLOBAL ARGUMENTS-CONTROL GRAPH

We collect the relevant information under the form of an arguments-control graph. Let $X \in \{CON, PRO\}$. If $X = PRO$ (resp. CON), $\overline{X} = CON$ (resp. PRO).

- constructive control: $X^{+}_{(a,b)}$ iff $|add_{(a,b)} \cap X| > |rem_{(a,b)} \cap \overline{X}|$
  the number of agents in X who can add (a, b) is greater than the number of agents in $\overline{X}$ who can remove it.
- destructive control: $X^{-}_{(a,b)}$ iff $|rem_{(a,b)} \cap X| \geq |add_{(a,b)} \cap \overline{X}|$
  the number of agents in X who can remove (a, b) is greater or equal than the number of agents in $\overline{X}$ who can add it.
- playability: $X_{\bullet}$ iff $|add_{(a,b)} \cap X| > 0$
  the move can be played.

Properties


a
b
d
c
agent 1
a
b
d
c
agent 2
a
b
d
c
agent 3
a
b
d
c
agent 4
a
b
d
c
agent 5


a
$\text{CON}_\bullet^{+,-}, \text{PRO}_\bullet$
b
$\text{CON}_\bullet^{+}, \text{PRO}_\bullet^{+}$
d
$\text{CON}_\bullet^{-}, \text{PRO}^{-}$
$\text{CON}_\bullet^{+}$
$\text{PRO}_\bullet^{+}$
$\text{CON}_\bullet^{-}, \text{PRO}^{-}$
c

WHO WINS THE DEBATE?

An edge (a, b) is an attack (resp. defense) edge for d if there is an even (resp. odd) length path from b to d. Note that an edge may be both attack and defense.

Definition

A **path for** d **controlled by** CON is an odd-length path from x to d such that (i) CON has constructive control on all the attack edges for d, and (ii) CON has destructive control on all the defense edges for d attacking x.

Possible outcome for CON

The issue d is a possible outcome for CON if there exists a **tree** for d controlled by CON.

WHO WINS THE DEBATE?

However the issue may be possible for PRO even when such paths exist. Intuitively, some moves that are both attack/defense moves can block the paths controlled by CON.

Definition

An edge (x, y) on a path P is a **switch** for d if (i) it is a defense for d on P, (ii) it is playable by *CON*, (iii) there exists a even-length path from y to d such that all the attack edges are playable by *CON* and all the defense edges are playable by *PRO*, and PRO has the destructive control on at least one attack edge from this path.

COINCIDENCE WITH THE MERGED OUTCOME?

In general, no guarantee that the outcome will be similar to the one obtained via merging. Can we find sufficient conditions for this to hold?

Reachability of the merged outcome

When the ACG does not contain edge (a, b) such that $X^{+,-}(a, b)$, then *reachability* of the merged outcome is guaranteed.

Protocol with reinforcement moves?

Idea: allow agents to reinforce (resp. weaken) moves that would change the status of the issue if it was to be deleted.

Ineffiency of reinforcement moves

It is never beneficial for an agent to play reinforcement moves (if weakening are symmetrically allowed). Furthermore, it may be the case that the issue is no longer a possible outcome if X uses reinforcement moves.

The key observation is to note that by playing reinforcement moves a group of agent may *lose* some destructive control.

ET AUSSI...

- multi-agent resource allocation
- abduction distribuée
- explanation de résultats de procédures de vote