Some ideas and open problems in real-time stochastic scheduling

Liliana CUCU, TRIO team, Nancy, France
Real-time systems

- Reactive systems
- Correct reaction
- Temporal constraints
Real-time systems (2)
Real-time systems (2)
Real-time systems (2)
Real-time systems (2)
Real-time systems (2)
Real-time systems (2)
Real-time systems (2)
Real-time model: \[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ \tau_1 = (O_1, C_1, T_1, D_1) = (1, 2, 5, 4) \]

release times

deadlines
Real-time model:

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ \tau_1 = (O_1, C_1, T_1, D_1) = (1, 2, 5, 4) \]
Real-time model:

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ \tau_1 = (O_1, C_1, T_1, D_1) = (1, 2, 5, 4) \]

\[ \geq T_1 \]

response times

deadlines
Why stochastic?

- Soft real-time constraints
- Uncertainty
- Worst-case behavior is a rare event
Where is the “stochastic touch”? 
Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions
Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions and
Where is the “stochastic touch”?  

Extracting quantitative information, i.e., obtaining distribution functions  

and  

Temporal analysis of systems with at least one parameter given by a random variables
Where is the “stochastic touch”? 

Extracting quantitative information, i.e., obtaining distribution functions

and

Temporal analysis of systems with at least one parameter given by a random variables
Extracting quantitative information

Data → Distribution functions
Extracting quantitative information

Data → ? worst-case behaviour are rare events → Distribution functions
Extracting quantitative information

Joint work with N. Navet and René Schott (TRIO, Nancy)
How to estimate the average response time???
How to estimate the average response time???

Activation model of tasks not known
How to estimate the average response time???

Activation model of tasks not known

Monte-Carlo simulation
How to estimate the average response time???

Activation model of tasks not known

- Monte-Carlo simulation
- Analytical approaches
How to estimate the average response time???

Activation model of tasks not known

Monte-Carlo simulation
Analytical approaches
Markov’s, Tchebychev’s, Chemoff’s upper bounds
How to estimate the average response time???

Activation model of tasks not known

- Monte-Carlo simulation
- Analytical approaches
- Markov’s, Tchebychev’s, Chemoff’s upper bounds
- Large deviation
How to estimate the average response time???

Activation model of tasks not known

- Monte-Carlo simulation
- Analytical approaches

Markov’s, Tchebychev’s, Chemoff’s upper bounds

- better suited than simulation to rare events
- easily implementable
- embedded in a broader analysis
Large deviation : main result

\[ M_n = \frac{1}{n} \sum_{k=1}^{n} R_{i,k} \] mean of response times over \( n \) task instances

\[ P(M_n \geq \text{value}) \]

Cramer’s theorem: if \( R_{i,n} \) independent identically distributed random variables

\[ P(M_n \in G) \approx e^{-n \inf_{x \in G} I(x)} \]

\[ G = [\text{value}, \infty) \]

\[ I(x) = \sup_{\tau > 0} [\tau x - \log E(e^{\tau x})] = \sup_{\tau > 0} [\tau x - \log \sum_{k=-\infty}^{+\infty} p_k e^{k\tau}] \]
Technical contribution

Can deal with distributions given as histograms

RT intervalProbability $k$

\[
\begin{array}{ccc}
[0, 10) & 1/25 & 5 \\
[10, 20) & 2/25 & 15 \\
[20, 30) & 3/25 & 25 \\
[30, 40) & 10/25 & 35 \\
[40, 50) & 4/25 & 45 \\
[50, 60) & 3/25 & 55 \\
[60, 70) & 2/25 & 65 \\
\end{array}
\]
Technical contribution

Can deal with distributions given as histograms

RT intervalProbability $k$

<table>
<thead>
<tr>
<th>Interval</th>
<th>Probability</th>
<th>$k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0, 10)$</td>
<td>1/25</td>
<td>5</td>
</tr>
<tr>
<td>$[10, 20)$</td>
<td>2/25</td>
<td>15</td>
</tr>
<tr>
<td>$[20, 30)$</td>
<td>3/25</td>
<td>25</td>
</tr>
<tr>
<td>$[30, 40)$</td>
<td>10/25</td>
<td>35</td>
</tr>
<tr>
<td>$[40, 50)$</td>
<td>4/25</td>
<td>45</td>
</tr>
<tr>
<td>$[50, 60)$</td>
<td>3/25</td>
<td>55</td>
</tr>
<tr>
<td>$[60, 70)$</td>
<td>2/25</td>
<td>65</td>
</tr>
</tbody>
</table>

!! Uniprocessor or multiprocessor !!

Number of successive task instances ($n$)

Upper bound on the probability

Gotha-- Liliana CUCU - 04/04/2008
Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions

and

Temporal analysis of systems with at least one parameter given by a random variables
Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions

and

Temporal analysis of systems with at least one parameter given by a random variables
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \left( \begin{array}{c} x_k \\ P(X = x_k) \end{array} \right) \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \left( \begin{array}{c} x_k \\ P(X = x_k) \end{array} \right) \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \left( \frac{x_k}{P(X = x_k)} \right) \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \left( \begin{array}{c} x_k \\ P(X = x_k) \end{array} \right) \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix} \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \left( \begin{array}{c} x_k \\ P(X = x_k) \end{array} \right) \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix} \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix} \]
What is the model?

\[ \tau_i = (O_i, C_i, T_i, D_i) \]

\[ X = \left( \begin{array}{c} x_k \\ P(X = x_k) \end{array} \right) \]
What do we want?

Response time \( R_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix} \)
What do we want?

Response time \( R_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix} \)

Satisfied deadline \( satisfyDeadline_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix} \)
Response time $\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$

Satisfied deadline $satisfyDeadline_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix}$

Response time jitter $J_i = \begin{pmatrix} 2 & 3 & 4 \\ 0.7 & 0.1 & 0.3 \end{pmatrix}$

etc ...
What do we want?

Response time \( R_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix} \)

Satisfied deadline \( satisfyDeadline_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix} \)

Response time jitter \( J_i = \begin{pmatrix} 2 & 3 & 4 \\ 0.7 & 0.1 & 0.3 \end{pmatrix} \)

Simulations? Analytical proofs? etc ...
What do we want?

Response time \( R_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix} \)

Satisfied deadline \( satisfy\text{Deadline}_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix} \)

Response time jitter \( J_i = \begin{pmatrix} 2 & 3 & 4 \\ 0.7 & 0.1 & 0.3 \end{pmatrix} \)

Simulations? Analytical proofs? etc ...

Joint work with E. Tovar (Hurray, Portugal)
Response time of a task $\tau_i$

When minimal inter-arrival times are considered

$$R_i = C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$
Response time

\[ R_i = C_i \otimes \left( \otimes_{k \in P} \frac{R_i}{C_k} \right) \otimes \left( \otimes_{k \in RN}^{\tau_k} C_k \right) \]
Algorithm providing a solution

Initial value \( R^0_i = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad N_{\tau_k} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \forall k \in P_{hp(i)} \)

New arrivals are added to the response time

Response time contains only changed values

All values unchanged or deadline missed?

NO

YES

SOLUTION

\[ R_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix} \]
Initial values

\[ R_i^0 = \left( \begin{array}{c} r_{i,1}^0 \\ 1 \end{array} \right) = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \] and \[ N_k = \left( \begin{array}{c} 0 \\ 1 \end{array} \right), \forall k \in R_{hp(i)} \]
Iteration $m$ - first step

Working random variable $L^m$

\[
L^m_j = C_i + \sum_{k \in P_{hp(i)}} \left[ \frac{L^m_j}{T_k} \right] \cdot C_k + \sum_{k \in R_{hp(i)}} N_k(r^m_{j-1}) \cdot C_k
\]

$r^m_{j-1}$ initial value
An example

<table>
<thead>
<tr>
<th>Task</th>
<th>T</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ₁</td>
<td>$T_1 = \begin{pmatrix} 8 &amp; 10 &amp; 15 \ 0.1 &amp; 0.3 &amp; 0.6 \end{pmatrix}$</td>
<td>3</td>
</tr>
<tr>
<td>τ₂</td>
<td>$T_2 = \begin{pmatrix} 10 \ 1 \end{pmatrix}$</td>
<td>3</td>
</tr>
<tr>
<td>τ₃</td>
<td>$T_3 = \begin{pmatrix} 15 &amp; 20 \ 0.6 &amp; 0.4 \end{pmatrix}$</td>
<td>2</td>
</tr>
<tr>
<td>τ₄</td>
<td>$T_4 = \begin{pmatrix} 15 \ 1 \end{pmatrix}$</td>
<td>2</td>
</tr>
<tr>
<td>τ₅</td>
<td>$T_5 = \begin{pmatrix} 14 &amp; 22 \ 0.4 &amp; 0.6 \end{pmatrix}$</td>
<td>2</td>
</tr>
</tbody>
</table>

$L_4^1 = \begin{pmatrix} l_1^1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ where $l_1^1$ solution of equation $l_1^1 = 0 \cdot C_1 + \left[ \frac{l_1^1}{T_2} \right] C_2 + 0 \cdot C_3 + 1 \cdot C_4$

with $r_1^0 = 0$ initial value
Iteration $m$ - second step

\[ \mathcal{R}_i^m = L^m \bigotimes \left( \bigotimes_{k \in R_{hp}(i)} \Delta_k \cdot C_k \right) \]
Back to the example

<table>
<thead>
<tr>
<th>Task</th>
<th>T</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
<td>( T_1 = \begin{pmatrix} 8 &amp; 10 &amp; 15 \ 0.1 &amp; 0.3 &amp; 0.6 \end{pmatrix} )</td>
<td>3</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>( T_2 = \begin{pmatrix} 10 \ 1 \end{pmatrix} )</td>
<td>3</td>
</tr>
<tr>
<td>( \tau_3 )</td>
<td>( T_3 = \begin{pmatrix} 15 &amp; 20 \ 0.6 &amp; 0.4 \end{pmatrix} )</td>
<td>2</td>
</tr>
<tr>
<td>( \tau_4 )</td>
<td>( T_4 = \begin{pmatrix} 15 \ 1 \end{pmatrix} )</td>
<td>2</td>
</tr>
<tr>
<td>( \tau_5 )</td>
<td>( T_5 = \begin{pmatrix} 14 &amp; 22 \ 0.4 &amp; 0.6 \end{pmatrix} )</td>
<td>2</td>
</tr>
</tbody>
</table>

\[ \mathfrak{A}_4^2 = L^2 \otimes \begin{pmatrix} 1 & 2 \\ 0.6 & 0.4 \end{pmatrix} C_1 \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} C_3 \]
Iteration $m$ - get ride of unchanged values

\[
L^m = \begin{pmatrix}
1 & 3 & 4 \\
0.5 & 0.2 & 0.3
\end{pmatrix}
\]

\[
\mathcal{R}_i^m = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 \\
0.1 & 0.4 & 0.3 & 0.1 & 0.1
\end{pmatrix}
\]

New $\mathcal{R}_i^m = \text{Comp}(\mathcal{R}_i^m, L^m) = \begin{pmatrix}
2 & 5 \\
0.4 & 0.1
\end{pmatrix}$
One entire iteration (3) of our example

The periodic higher tasks are giving a response time:

\[ I = \begin{pmatrix} 20 \\ 0.5 \end{pmatrix} \]

The random higher tasks are giving a response time:

\[ R_{n,0}^3 = I \otimes (F^*(20) \cdot C_3) = \begin{pmatrix} 20 & 21 \\ 0.42 & 0.08 \end{pmatrix}, \]

where \( F^*(20) = \begin{pmatrix} 2 & 3 \\ 0.84 & 0.16 \end{pmatrix} \)
One entire iteration (3) of our example

The periodic higher tasks are giving a response time:

\[ I = \begin{pmatrix} 20 \\ 0.5 \end{pmatrix} \]

The random higher tasks are giving a response time:

\[
\mathcal{R}^3_{n,0} = I \otimes (F^*(20) \cdot C_3) = \begin{pmatrix} 20 & 21 \\ 0.42 & 0.08 \end{pmatrix},
\]

where \( F^*(20) = \begin{pmatrix} 2 & 3 \\ 0.84 & 0.16 \end{pmatrix} \)
Some precautions when we think stochastic ...
Some precautions when we think stochastic ...

Analysis able to give an answer in the deterministic case and to allow mixing hard and soft real-time constraints
Some precautions when we think stochastic...

☑️ Analysis able to give an answer in the deterministic case and to allow mixing hard and soft real-time constraints
☑️ Robustness based on large deviations
Some precautions when we think stochastic ...

- Analysis able to give an answer in the deterministic case and to allow mixing hard and soft real-time constraints
- Robustness based on large deviations
- Next step?
How to validate stochastic?
How to validate stochastic?

Initial condition: deterministic case
How to validate stochastic?

- Initial condition: deterministic case
- Robustness condition: worst-case behavior of the algorithms is rare
How to validate stochastic?

- Initial condition: deterministic case
- Robustness condition: worst-case behavior of the algorithms is rare
- Worst-case condition: insure worst-case when mixing hard and soft
Some references

A. Burns, G. Bernat, I. Broster - A probabilistic framework for schedulability analysis
- Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview
- Statistical analysis of WCET for scheduling
- Probabilistic estimation of response times through large deviations
- Stochastic analysis of real-time systems

M. Pinedo

S. Edgar, A. Burns
N. Navet, L. Cucu, R. Schott

L. Cucu, E. Tovar

A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times
### Some references

<table>
<thead>
<tr>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Burns, G. Bernat, I. Broster</td>
<td>A probabilistic framework for schedulability analysis</td>
</tr>
<tr>
<td>M. Pinedo</td>
<td>Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview</td>
</tr>
<tr>
<td>S. Edgar, A. Burns, N. Navet, L. Cucu, R. Schott</td>
<td>Statistical analysis of WCET for scheduling</td>
</tr>
<tr>
<td></td>
<td>Stochastic analysis of real-time systems</td>
</tr>
<tr>
<td></td>
<td>A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times</td>
</tr>
</tbody>
</table>
### Some references

<table>
<thead>
<tr>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Burns, G. Bernat, I. Broster</td>
<td>A probabilistic framework for schedulability analysis</td>
</tr>
<tr>
<td>M. Pinedo</td>
<td>Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview</td>
</tr>
<tr>
<td>S. Edgar, A. Burns</td>
<td>Statistical analysis of WCET for scheduling</td>
</tr>
<tr>
<td>N. Navet, L. Cucu, R. Schott</td>
<td>Probabilistic estimation of response times through large deviations</td>
</tr>
<tr>
<td>L. Lo Bello, J.M. Lopez, S.L. Min, O.</td>
<td>A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times</td>
</tr>
<tr>
<td>Mirabella</td>
<td></td>
</tr>
<tr>
<td>L. Cucu, E. Tovar</td>
<td></td>
</tr>
</tbody>
</table>
**Some references**

<table>
<thead>
<tr>
<th>Authors</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Burns, G. Bernat, I. Broster</td>
<td>A probabilistic framework for schedulability analysis</td>
</tr>
<tr>
<td>M. Pinedo</td>
<td>Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview</td>
</tr>
<tr>
<td>S. Edgar, A. Burns, N. Navet, L. Cucu, R. Schott</td>
<td>Statistical analysis of WCET for scheduling</td>
</tr>
<tr>
<td></td>
<td>Probabilistic estimation of response times through large deviations</td>
</tr>
<tr>
<td></td>
<td>A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times</td>
</tr>
</tbody>
</table>
Open problems in stochastic real-time scheduling: introduction of dependent random variables.