Two Phase Stochastic Local Search Algorithms for the Biobjective Traveling Salesman Problem

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Abstract

In this work, we present two phase stochastic local search algorithms with the aim of finding a good approximation of the efficient solution set of the biobjective traveling salesman problem. In the first phase of the algorithms, a search for a good approximation of the supported efficient solution set is undertaken. After this first phase, the second phase is launched to generate non-supported efficient solutions. Three methods are presented and experimented for the second phase: Pareto local search, a memetic algorithm with a data perturbation technique and a path-relinking operator.

1 The mTSP

Given a set $\{v_1, v_2, \cdots, v_N\}$ of cities and K costs $c_k(v_i, v_j)$ (with $k = 1, \ldots, K$) between each pair of distinct cities $\{v_i, v_j\}$ (with $i \neq j$), the multiobjective traveling salesman problem (mTSP) consists of finding a solution, i.e. an order π of the cities, so as to minimize the following costs ($k = 1, \ldots, K$):

$$\text{“}\min\text{”} z_k(\pi) = \sum_{i=1}^{N-1} c_k(v_{\pi(i)}, v_{\pi(i+1)}) + c_k(v_{\pi(N)}, v_{\pi(1)})$$

These K quantities z_k correspond to the values taken by the various objectives, for a tour realized by a traveling salesman who visits each city exactly once and then returns to the starting city. We are interested here only in the symmetric biobjective traveling salesman problem (bTSP), i.e. $c_k(v_i, v_j) = c_k(v_j, v_i)$ for $1 \leq i, j \leq N$ and $K = 2$.

Due to the contradictory features of the objectives, it does not exist a solution simultaneously minimizing each objective (and for this reason the notation "min" is used), but a set of solutions called *efficient solutions*. A solution π^* is efficient for the mTSP if there is no other solution π such that: $z_k(\pi) \leq z_k(\pi^*), k = 1, \ldots, K$ with at least one strict inequality.

In this paper, only a minimal complete set will be sought, i.e. no equivalent efficient solution (two solutions π_1 and π_2 are equivalent if $z_k(\pi_1) = z_k(\pi_2)$, $k = 1, \ldots, K$) will be retained, and each solution found will correspond to a distinct non-dominated point in the objective space. We call this minimal complete set *Pareto set*.

2 Solution methods

Given the difficulty of the bTSP, we only try to find a good approximation of the Pareto set. Three different stochastic local search algorithms are experimented that are all based on the same two phases [8]:

1. Phase 1: Find a good approximation of the supported efficient solution set (solutions whose objective vectors lies in the border of the convex-hull of the Pareto set). These solutions can be obtain by resolution of single-objective problems obtained by applying a linear aggregation of the objectives: $\sum_{i=1}^{K} \lambda_k z_k(\pi)$ where λ is a weight set, i.e. a vector of dimension K, with $0 \leq \lambda_k \leq 1$ for $k = 1, \ldots, K$ and $\sum_{k=1}^{K} \lambda_k = 1$.

2. Phase 2: Find the non-supported efficient solutions (solutions not lying in the border of the convex-hull) located between the supported efficient solutions.

2.1 Approximation of the supported efficient solution set

We employ the method of Aneja and Nair [1], initially proposed for the resolution of a bicriteria transportation problem, that consists in generating all the weight sets which make it possible to obtain a minimal complete set of extremal supported efficient solutions (solutions whose objective vectors are located on the vertex set of the convex-hull) of a biobjective problem (non-extremal supported efficient solutions and equivalent solutions can however be generated). For each weight set generated, a linear aggregation of the objectives is carried out and the single-objective problem obtained is solved by an exact method. In this work, we do not use an exact method to solve the single-objective problem but the Lin-Kernighan (`LK`) heuristic implemented by Helsgaun [3]. This heuristic gives for the instances of 100 cities studied in this work very good solutions, close to the optimal solutions.

So, we have adapted the method of Aneja and Nair to take into account the fact that the `LK` heuristic is not exact, what implies that the solutions obtained are not necessarily efficient, nor supported efficient but that makes it possible to obtain a set of solutions very close to the minimal complete set of extremal supported efficient solutions, with a minimum number of resolution of single-objective problems resulting from linear aggregation.

2.2 Search for non-supported efficient solutions

Once a good approximation of the supported efficient solution set has been found, three methods are experimented with the aim of finding potentially non-supported efficient solutions. These three methods all use an archive containing the potentially efficient solutions found, which is updated as soon as a new potentially efficient solution is discovered, by adding the new solution in the archive and by removing the solutions of the archive which could be found dominated following the addition of the new solution. After the phase 1, the archive contains, for all the methods, an approximation of the supported efficient solution set.

2.2.1 Pareto local search

This method has been developed by Paquete et al. [6] and is based on the notion of *Pareto local optimum set* which is a generalization, in the multi-objective case, of the concept of local optimum. In this method, the neighborhood of each solution of the archive is explored, and each non-dominated neighbor is added to the archive. The algorithm stops when it is any more possible to find new non-dominated neighbors starting from a solution of the archive, that is to say, a Pareto local optimum set is found, which respect to the neighborhood used. In this work, we use the well-known 2-exchange neighborhood, also used by Paquete et al. However, they start their method from a randomly generated solution, whereas we use all the solutions of the archive generated in phase 1 as initial solutions. We call this method `PLS2`.

2.2.2 Memetic algorithm

We use `MEMOX` [5], scheme of resolution of multiobjective problems, based on a memetic algorithm. After the phase 1, a local search is applied from an offspring solution, generated by a crossover between a solution of the archive of minimal density and an another solution of the archive close, in the objective space, of the first solution. A dynamic hypergrid is used to compute the density of a potentially efficient solution. We use the `LK` heuristic as local search, by employing a linear aggregation of the objectives with a weight set fixed according to the first parent, i.e. the potentially efficient solution of minimal density. But, as the `LK` heuristic is very robust (very little influenced by the starting solution), few new solutions will be found by the local search based on a linear aggregation, since a search for the supported efficient solutions has already been applied during the phase 1. We thus use the *Data Perturbation* (`DP`) technique, originally proposed by Codenotti et al. for the single-objective TSP [2]. Instead of modifying the starting solution (as carried out, for example, in the Iterated Local Search method), DP suggests to modify input data. In this way, by application of the LK heuristic starting from the offspring with perturbed data, new solutions, essentially potentially non-supported efficient, could be found since the data used for the linear aggregation are perturbed.

2.2.3 Path-relinking

Before applying the path-relinking (PR), the solutions of the archive generated in phase 1 are sorted according to the increasing order of the value taken by the first objective. Then, a path in the decision space between two consecutive solutions of the archive, called starting and guiding solutions, is created, with the goal of providing new solutions that reduce the distance with respect to the guiding solution, on the basis of the starting solution. A distance between two solutions is measured by the number of uncommon arcs in both solutions. The 2-exchange movement is used to create the path, and only movements that reduce the distance are considered. Among such movements, the one that generates the nearest solution in the objective space to the line which connects the starting and the guiding solution is selected. Every new non-dominated solution found during the path building process is added to the archive.

3 Results

First experimentations show that compared to the state-of-the-art algorithms (MOGLS [4], PLS [6], PD-TPLS [7]) the results obtained by these three methods on instances of 100 cities of the bTSP are of better qualities. Using PLS2 is very efficient and allows to obtain very good approximations in a reasonable time. The use of an initial archive of good quality (generated in phase 1) is clearly better than using as first archive only one randomly generated solution as done by Paquete et al. in [6]. Applying PR as phase 2 gives good results in little time, but of lower quality than using PLS2. Moreover, if we try to improve the results of PR by applying a Pareto local search on the archive obtained, the results are not better than PLS2. The disadvantages of PR and PLS2 are that their performance is limited, and more computational time will not give significant better results, being given that these two methods are limited by the quality of the 2-exchange neighborhood. On the other hand, although the MEMOX scheme with the LK heuristic as local search with perturbed data converges more slowly than the other methods, this method allows to obtain better results if the resolution time is increased, since thanks to the data perturbations, new solutions are constantly found what increases the quality of the solution set obtained.

4 Conclusion

Work still needs to be done to take advantage of each of the three methods used for the search of non-supported solutions, essentially by allowing perturbations in the PLS method to avoid being stuck in a Pareto local optimal set. The perturbations can be, for example, realized by the data perturbation technique, the path-relinking operator or by allowing to have dominated solutions in the archive.

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