MULTICRITERIA MAINTENANCE PROBLEM RESOLVED BY TABU SEARCH

Lust Thibaut * **Teghem Jacques** *

* *Laboratory of Mathematics & Operational Research*
Faculté Polytechnique de Mons
9 rue de Houdain, B-7000 Mons, Belgium

Abstract: We first present in this paper the multicriteria case of the selective maintenance problem. This problem consists in finding the best choice of maintenance actions to realize on a multicomponent system, so as to maximize the system reliability and minimize the maintenance cost, but with a limitation in the maintenance time. Since exact methods are inefficient when a lot of components compose the system, we applied a new general method, based on tabu search, for the resolution of multicriteria combinatorial optimization problems. The aim is to quickly generate a good approximation of the whole set of efficient solutions.

Keywords: Preventive Maintenance, Reliability, Multicriteria Optimization, Efficient solutions, Tabu Search.

1. INTRODUCTION

In this paper we are interested in preventive maintenance, and more particularly in the selective maintenance problem, which consists in finding the best choice of maintenance actions to realize on a multicomponent system, but with a limitation in the maintenance time, so that all maintenance actions can't be carried out. Applications include all equipment which perform sequences of missions and are repaired only between missions: military equipment, production equipment which maintenance actions are carried out the week-end, vehicles maintained between two deliveries, etc. This problem has been introduced by Rice et al. (Rice *et al.*, 1998), where they consider systems presenting a particular architecture, constant components failure probabilities and only one type of maintenance actions (to repair a component). Cassady et al. extended the model of this problem, by considering components failure probabilities dependent on their age and multiple maintenance actions (Cassady *et al.*, 2001). They both consider only one criterion, maximizing the system reliability. We thus extend this problem to the multicriteria case, by adding one important criteria: minimizing the maintenance cost. Since these two criteria are contradictory, the problem presents multiple solutions, correspondent to different compromises.

In the unicriteria case, the resolution method used by Cassady et al. is a simple enumeration of all possible solutions, which can only work for systems of small size. We proposed recently a heuristic and a branch and bound procedure (Lust *et al.*, 2006) to reduce the resolution time, but which can also only work in the uncriteria case.

We therefore apply the `PRTS+D` (Pareto `R`anking `T`abu `S`earch + `D`ensity) method (Lust and Teghem, 2006), based on tabu search. This method approximates the whole set of efficient solutions of multicriteria combinatorial problems, and doesn't use any aggregation functions based on weight sets. It seems to be a real advantage in comparison with other multicriteria methods like MOSA (Ulungu *et al.*, 1999) or MOGLS (Jaszkiewicz, 2002), since the use of an aggregation function requires an effective management, not always obvious, of weight sets, but also a normalization of the values taken by the criteria, which demands the knowledge of the bounds in which they are located.

This paper is organized as follows. We first present the multicriteria selective maintenance problem, and then the PRTS+D method. We finally apply the method to a study case.

2. THE MULTICRITERIA SELECTIVE MAINTENANCE PROBLEM

We consider that maintenance actions are carried out on a system, defined by a set of components connected to each other in series and/or parallel. An example of such a system is presented at figure 1. In this representation, the blocks correspond to components. The failure of one of the components 3,4,5 or 10 placed in series involves the system failure. Components 6,7,8 or 9 being placed in parallel, the system fails only if the two parts of the parallel subsystem aren't functioning. That arrives when the failure of one of the components 6 or 7 is in combination with the failure of one of the components 8 or 9.

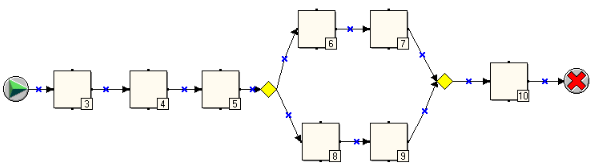

Fig. 1. Example of multicomponent system.

The system must perform a sequence of missions with breaks of known length between each mission, while the maintenance actions are accomplished. During each mission, components can fail. Consequently, at the end of a mission, the components are either in operating condition or failed. As Cassady et al., we consider two possible maintenance actions during the break (Cassady *et al.*, 2001):

- to replace (by new) a failed or a functioning component (the component age after the action is thus equal to 0),
- to minimally repair a failed component, what restart the component (the component age after the action is thus unchanged).

Each maintenance action spends time, has a cost, but makes it possible to increase the system reliability for a mission of a certain duration, defined by the probability that it achieves the mission. The bicriteria problem considered is to find the actions maximizing the system reliability and minimizing the cost, under time constraint. For example, we can have maximum two days to carry out the maintenance actions.

We consider that the component failure probability of a component for a given mission is dependent on its age, i.e. the higher the component age is, the higher its failure probability is. This probability is modeling through a reliability law, which is in fact a probability law. Most current in maintenance is the Weibull distribution. Thanks to the components probabilities, we can determine the functioning probability of a system for a mission of a given duration, which depends on the functioning probabilities of the components and on the system architecture. Contrary to preceding modelings of the selective maintenance problem (Rice *et al.*, 1998; Cassady *et al.*, 2001), we consider in this paper any system in series and/or parallel.

We present below how to calculate the functioning probability of a component thanks to its Weibull distribution as well as the functioning probability of a system according to the functioning probabilities of the components which compose it.

2.1 Functioning probability of a component

The reliability law gives the probability that the component achieves without failure a mission of length t. In the case of the Weibull distribution, the probability $R(t)$ is given by the following relation:

$$R(t) = e^{-(\frac{t}{\eta})^{\beta}}$$

where β and η represent the shape and scale parameters of the Weibull distribution. They are real numbers superiors to zero.

If the component already carried out a mission of length T and is in operating condition at the end of this mission, we use conditional reliability to determine the probability that the component achieves successfully a new mission of length t, defined by the following relation:

$$R(T,t) = \frac{R(T+t)}{R(T)} = \frac{e^{-(\frac{T+t}{\eta})^{\beta}}}{e^{-(\frac{T}{\eta})^{\beta}}}$$

Hence, starting from the component age and the mission duration, it's easy, in the case of the Weibull distribution, to determine the functioning probability of a component.

2.2 Functioning probability of a multicomponent system

We can break up a multicomponent system into a set of subsystems entirely in series or parallel. The

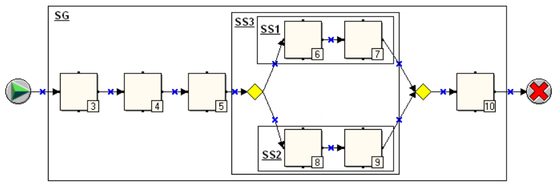

Fig. 2. Example of decomposed multicomponent system.

decomposition of the system of figure 1 is given at figure 2. We have subsystem 1 in series composed of components 6 and 7 (SS1), subsystem 2 in series composed of components 8 and 9 (SS2) and subsystem 3 in parallel composed of subsystems 1 and 2 (SS3). The global multicomponent system (SG) in series is thus composed of components 3,4,5,10 and subsystem 3. By using this decomposition, it is only necessary to determine the functioning probabilities of systems entirely in series ($\prod R_i(t)$), or functioning probabilities of systems entirely in parallel ($1-(1-\prod R_i(t))$).

3. PROBLEM MODELING

The modeling of the selective maintenance problem has already been approached by Cassady et al. (Cassady *et al.*, 2001). We take again partly this modeling by adding the new cost criteria, and by extending it to any system in series and/or parallel.

We indicate by tmr_i and cmr_i the time and cost necessary to carry out a minimal repair on a failed component i; tr_i and cr_i the time and cost necessary to replace a failed component and trf_i and crf_i the time and cost necessary to replace a functioning component. We suppose that $tmr_i <= tr_i$, $cmr_i <= cr_i$, $trf_i <= tr_i$ and $crf_i <= cr_i$, i.e. time and cost to replace a failed component are higher or equal to time and cost of other actions.

The system is composed of n components, functioning or failed at the end of mission k, depending on the state of the binary variable $Y_i(k)$:

$$Y_i(k) = \begin{cases} 1 \text{ if the component } i \text{ is functioning at} \\ \quad \text{the end of mission } k, \\ 0 \text{ otherwise.} \end{cases}$$

The minimal repair action on a failed component i at the end of mission k is symbolized by the binary variable $W_i(k)$, so that:

$$W_i(k) = \begin{cases} 1 \text{ if minimal repair is realized} \\ \quad \text{between missions } k \text{ and } k+1, \\ 0 \text{ otherwise.} \end{cases}$$

In the same way, the action to replace a component i at the end of mission k is defined by the binary variable $V_i(k)$:

$$V_i(k) = \begin{cases} 1 \text{ if replacement is realized} \\ \quad \text{between missions } k \text{ and } k+1, \\ 0 \text{ otherwise.} \end{cases}$$

Total time $TMR(k)$ and total cost $CMR(k)$ necessary to minimal repairs are given by:

$$TMR(k) = \sum_{i=1}^{n} tmr_i \cdot W_i(k)$$

$$CMR(k) = \sum_{i=1}^{n} cmr_i \cdot W_i(k)$$

Total time $TR(k)$ and total cost $CR(k)$ necessary to replacements are composed of two parts, depending on the components state $Y_i(k)$:

$$TR(k) = \sum_{i=1}^{n} tr_i \cdot V_i(k) \cdot (1 - Y_i(k)) + \sum_{i=1}^{n} trf_i \cdot V_i(k) \cdot Y_i(k)$$

$$CR(k) = \sum_{i=1}^{n} cr_i \cdot V_i(k) \cdot (1 - Y_i(k)) + \sum_{i=1}^{n} crf_i \cdot V_i(k) \cdot Y_i(k)$$

We can thus express the total time $T(k)$ and total cost $C(k)$ necessary to the maintenance actions $W_i(k)$ and $V_i(k)$ at the end of a mission k by:

$$T(k) = TMR(k) + TR(k)$$

$$C(k) = CMR(k) + CR(k)$$

We also consider that the length of mission $k+1$ is equal to $L(k+1)$, that the maintenance time after the mission k is equal to $T_0(k)$, that $B_i(k)$ and $A_i(k+1)$ are respectively the component age after the mission k and before the mission $k+1$ and that $X_i(k+1)$ is the component state before the mission $k+1$. Each component follows a Weibull distribution of shape parameter β_i and scale parameter η_i. The system reliability $R(k+1)$ before the mission $k+1$ is given by the function F (dependent on the system architecture), which receives in argument the functioning probabilities of the components at the end of mission $k+1$. These probabilities $R_i(k+1)$ are given for each component i by the following relation:

$$R_i(k+1) = \frac{e^{-(\frac{L(k+1)+A_i(k+1)}{\eta_i})^{\beta_i}}}{e^{-(\frac{A_i(k+1)}{\eta_i})^{\beta_i}}}$$

The modeling obtained for the selective maintenance problem, which consists in determining which components to replace (decisions represented by $V_i(k)$ variables) and which components to minimally repair (decisions represented by $W_i(k)$ variables) at the end of mission k is given below. One can notice that we obtain a nonlinear multicriteria combinatorial optimization problem.

$$\left[\begin{array}{ll} \max R(k+1) & = F\big(R_i(k+1)\cdot X_i(k+1)\big) \\ \min \text{Cost} & = C(k) \\ \text{s.t} & T(k) \leq T_0(k) \\ & W_i(k)+V_i(k) \leq 1 \quad \forall\, i \\ & W_i(k)+Y_i(k) \leq 1 \quad \forall\, i \\ & W_i(k), V_i(k) \in \{0,1\} \quad \forall\, i \\ \text{with} & A_i(k+1) = B_i(k) - B_i(k)\cdot V_i(k) \quad \forall\, i \\ & X_i(k+1) = Y_i(k) + W_i(k) + \\ & \qquad V_i(k)\cdot(1-Y_i(k)) \quad \forall\, i \end{array}\right]$$

The problem presents four constraints: the first symbolizes the fact that the execution time of maintenance actions is limited, the second that only one of the two maintenance actions is carried out, the third that the action W_i is performed only if the component i is failed ($Y_i = 0$) and the fourth represents the $(0,1)$ integrity constraint. The two last equalities only make it possible to determine the components age and state after the maintenance break, necessary to the determination of the system reliability.

4. MUTICRITERIA TABU SEARCH: THE PRTS+D METHOD

The `PRTS+D` method follows the classical framework of tabu search (Glover and Laguna, 1998), but using an original evaluation function of neighbors, based on a recent paper of Elaoud, Loukil and Teghem (Elaoud *et al.*, 2005). They present a genetic algorithm (called `PFGA` for `P`areto `F`itness `G`enetic `A`lgorithm) using a new fitness function based on individuals rank and density. This approach giving promising results, the fitness function of `PFGA` has been adapted within a tabu search for the selection of the best neighbor.

Two measures operate in the fitness function of `PFGA`: an individuals classification according to the double pareto ranking (*DPR*) based on the dominance relation, and a division of the objectives space in hypervolumes allowing to measure the individuals density (*D*). These two concepts are then aggregate into a single evaluation function:

$$\frac{1}{e^{DPR(Y_l)} \times D(Y_l)}$$

We use this function for the choice of the best neighbor. The individuals for which one evaluates the aggregation function are thus the neighbors. The population includes the L neighbors generated at each iteration and the potentially efficient solutions found until then by the algorithm. The hypervolumes are created at each iteration according to the current population. The hypervolumes size, contrary to `PFGA`, are not of equal size, but follow a geometrical sequence, to favor individuals close to the Pareto front.

The selected neighbor is that which presents a maximum evaluation for the aggregation function defined above. Then we actualize the set of potentially efficient solutions *PE* by adding the non-dominated neighbors (identified by having a null value for the *DPR* rank) and by eliminating the potentially efficient solutions which could be found dominated.

The algorithm of the `PRTS+D` method is described at figure 3.

Parameters

m: tabu tenure
L: number of neighbors generated at each iteration (neighborhood size)
n: number of iterations
p: number of parts created along the objective axes
α: parameter of the geometrical sequence defining the parts length

Notations

$K \equiv$ number of criteria
$PE \equiv$ list of the non-dominated, potentially efficient solutions
$T \equiv$ tabu list
$X \equiv$ current solution
$N(X) \equiv$ neighborhood of X: set of feasible solutions obtained by a movement starting from X
$DPR(Y) \equiv$ double pareto ranking of a neighbor Y
$D(Y) \equiv$ density of a neighbor Y, defined by the number of solutions being in the same hypervolume as the neighbor

Initialization

Generate randomly a feasible solution X_0
$PE \leftarrow \{X_0\}$
$X \leftarrow X_0$
$T \leftarrow \emptyset$

Iteration i

Generate randomly L neighbors $Y_1, \ldots, Y_l, \ldots, Y_L$ not tabu in $N(X)$
Create p^K hypervolumes in the objective space
`For` each neighbor Y_l calculate $f(Y_l) = \frac{1}{e^{DPR(Y_l)} \times D(Y_l)}$
Calculate $f^* = f(Y_j{}^*) = \max_{l=1,\ldots,L} f(Y_l) \quad j = 1, \ldots, J$ (in case of ex æquo)
Choose randomly a neighbor Y^* among $\{Y_j{}^*, j = 1, \ldots, J\}$
$X \leftarrow Y^*$
Give to the movement $X \rightarrow Y^*$ the tabu statute
`For` each neighbor Y_l, `If` $DPR(Y_l) = 0$ `Then`
    $PE \leftarrow PE + \{Y_l\}$
    Actualize *PE*

Stop criterion

Iteration count $i = n$

Fig. 3. `PRTS+D` algorithm.

5. STUDY CASE PRESENTATION

The studied system is a refrigerating system coming from a production factory of liquified natural gas (LNG), presented by Smati et al. (Smati *et al.*, 2003).

As we can see at figure 4, the refrigerating system is composed of 24 components of 7 different types (balloon, pump, exchanger, valve, distributor, compressor, condenser) in series or parallel. The redundancy is strongly present. For example, two of the three subsystems composed of two condensers c can fail, without failure of the refrigerating system.

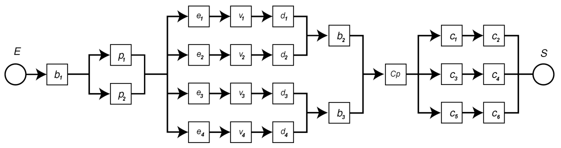

Fig. 4. Refrigerating system of liquified natural gas (LNG) process. b: separating balloons, p: cooling agent pumps, e: cooling agent exchangers, v: Joule-Thompson valves, d: distributors, Cp: axial compressor, c: see water condensers.

The components belonging to the same types are considered as identical. They have thus the same maintenance costs, maintenance times and reliability laws, all of Weibull type. The components characteristics are given at table 1.

The maximum time available $T_0(k)$ after the mission k for the maintenance actions realization is one weekend (48h). The system must carry out a mission $k+1$ of length $L(k+1)$ equal to 8 weeks. The system state before the intervention is given at figure 5. The components where a cross appears are the failed components ($Y_i(k) = 0$) at the end of mission k. The age B_i of components at this moment is indicated above each component (in days). The system is in operating condition but the probability that it achieves a mission of 8 weeks is low, since it's equal to 0.145. It's therefore necessary to determine which maintenance actions to realize, i.e. which components to minimally repair and which components to replace during the maintenance break. Since two contradictory criteria are considered, there is more than one solution and this is why we apply the PRTS+D method with the aim of finding a good approximation of the efficient solutions.

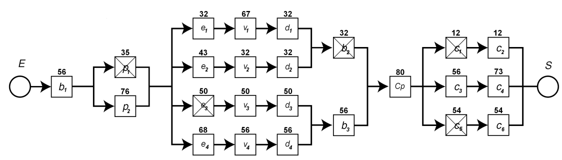

Fig. 5. State of the refrigerating system before the maintenance actions.

5.1 PRTS+D adaptation

We present here the main adaptations necessary to the resolution of the selective maintenance problem:

Solution coding A solution is coded with an array of size $2 \cdot n$ representing the maintenance actions carried out on the n components. The n first boxes, subscripted of 0 to $n-1$, correspond to the $W_i(k)$ actions and the n following, subscripted of n to $2n-1$, to the $V_i(k)$ actions. The figure 6 represents a solution for $n = 6$ components. We carry out the action W on the third and fifth components, and the action V on the second, fourth and sixth components.

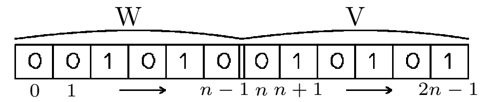

Fig. 6. Solution coding.

Starting solution definition The initial solution of the algorithm is simply generated by repairing all the failed components.

Neighborhood definition From the current solution X, we draw randomly an index z ranging between 0 and $2n-1$. If $0 \leq z \leq n-1$, that corresponds to an action W on the component z and if $n \leq z \leq 2n-1$ to an action V on the component $z-n$. The neighbor X' is obtained by doing or removing the action corresponding to the index z of the array of X, while being attentive of not carrying out an action W on a component in functioning state, nor carrying out an action V and one action W on the same component. In this last case, the neighbor is generated by permutation of the actions V and W.

Once the neighbor X' is generated, it also should be checked that the sum of the maintenance actions times of the neighbor doesn't exceed the maintenance break of length $T_0(k)$.

Management of the tabu list The movement is characterized by the index of the modified action(s). We then forbid during m (tabu tenure) iterations the choice of this(these) action(s) for the generation of a new neighbor.

5.2 Results

The parameters of PRTS+D are given at table 2. The number of iterations is fixed at 5000 since beyond there isn't any more improvement.

Table 2. Parameters of PRTS+D.

n	L	m	α	p
5000	24	7	0.7	10

The efficient solutions obtained by PRTS+D for the refrigerating system case is given at figure 7. These solutions were obtained in less than five seconds. For comparison, we also test a total enumeration algorithm. It gives the same solutions (and that proves that the solutions of PRTS+D are the efficient solutions) but in 45 minutes.

Table 1. Characteristics of components types.

Components	tmr(h)	tr(h)	trf(h)	cmr(€)	cr(€)	crf(€)	β	η(days)
Balloon	5	6	5	150	300	280	2.8	224
Pump	4	5	3	280	400	310	3.1	181
Exchanger	4	7	5	220	460	390	3.7	172
Valve	3	4	3	110	180	150	2.2	165
Distributor	4	5	4	150	220	190	1.9	161
Compressor	6	7	4	400	600	520	2.2	239
Condenser	5	6	4	160	310	270	4.8	192

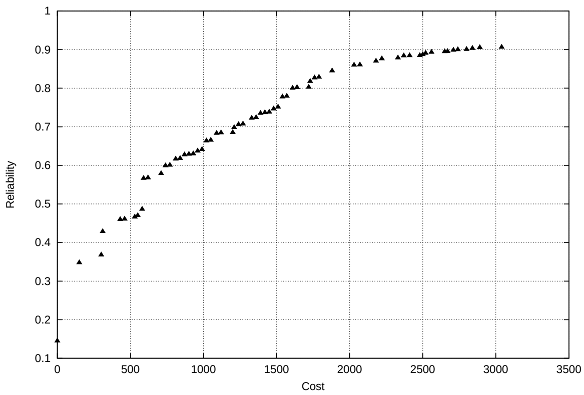

Fig. 7. Efficient solutions of the refrigerating system.

Moreover, this time grows exponentially with the complexity of the system.

One notes that one obtains 64 solutions which cost varies between 0 and 3040€ and reliability between 0.145 and 0.906. The manager is free to choose one of these solutions according to his preferences. For example, he will have to raise the question to know if he prefers to invest 1610€ and to obtain a reliability of 0.800 or to invest 1430€ more to obtain a reliability of 0.906.

At each one of these solutions corresponds maintenance actions to undertake. For example, the solution maximizing the reliability of the refrigerating system consists in replacing the first balloon b_1, the failed pump p_1, the failed exchanger e_3, the valves v_1 and v_3, the distributor d_3, the two balloons b_2 and b_3, the compressor and the failed condenser c_1. One notes that this solution doesn't consider the repair of one of the failed condensers, but privileges the replacement of critical components like the balloons and the compressor.

6. CONCLUSION

We presented in this paper the multicriteria selective maintenance problem. It has been solved thanks to the `PRTS+D` method. This method has the advantage of not using an aggregation function employing weight sets, which clear up the problem of their determination and the normalization of the values taken by the criteria. The original evaluation of the neighbors guarantees a good diversity and thus makes it possible to explore a great part of the objective spaces. The application of `PRTS+D` to a real problem, such as the multicriteria selective maintenance problem, is thus convincing and allowed to obtain the efficient solutions set of the study case.

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