Nash equilibria in Voronoi games on graphs

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this talk contains no random graphs, no approximation algorithms, no sugar

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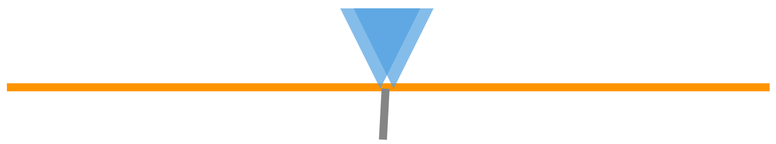
Ice vendors on a beach

[H.Hotteling, Stability in Competition, 1929]

a beach : a straight segment
tourists : buy from the closest vendor
gain of a vendor : surface of its Voronoi cell
social cost : average distance to the closest vendor

(unique) **social optimum**

(unique) pure Nash **equilibrium**



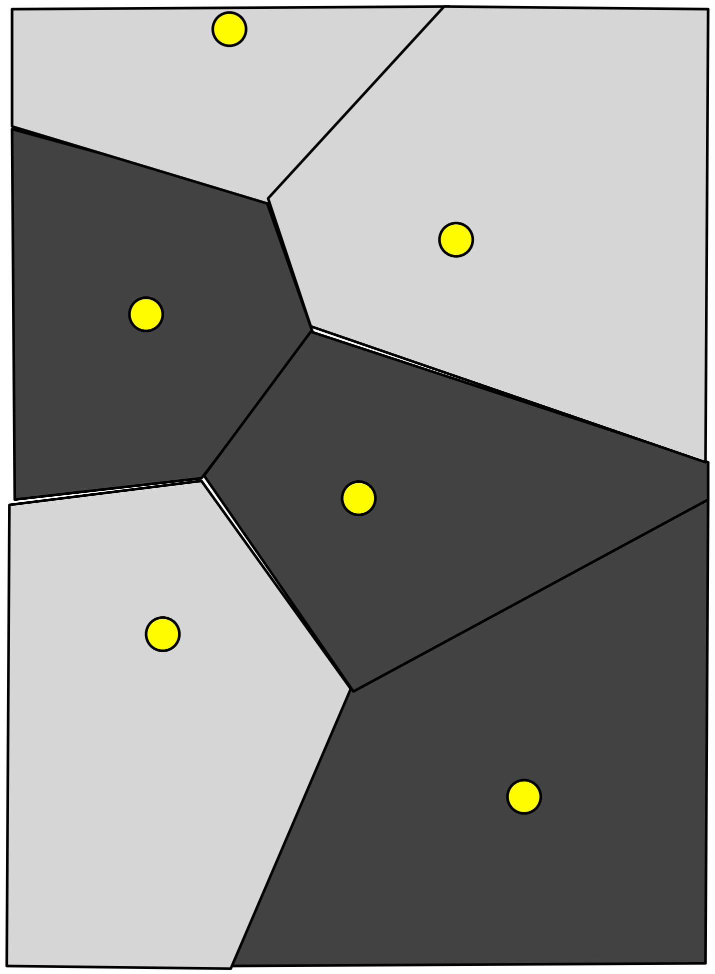
price of anarchy : (here 2) worst ratio between the social cost of an equilibrium and the social optimum

Past work on rectangles

- A rectangle of ratio $r \leq 1$, 2 players: White places p vendors, then black places p vendors.

  There is a strategy for black to win strictly more than half if and only if $p \geq 3$ et $r > \sqrt{2}/p$ or $p = 2$ and $r > \sqrt{3}/2$.

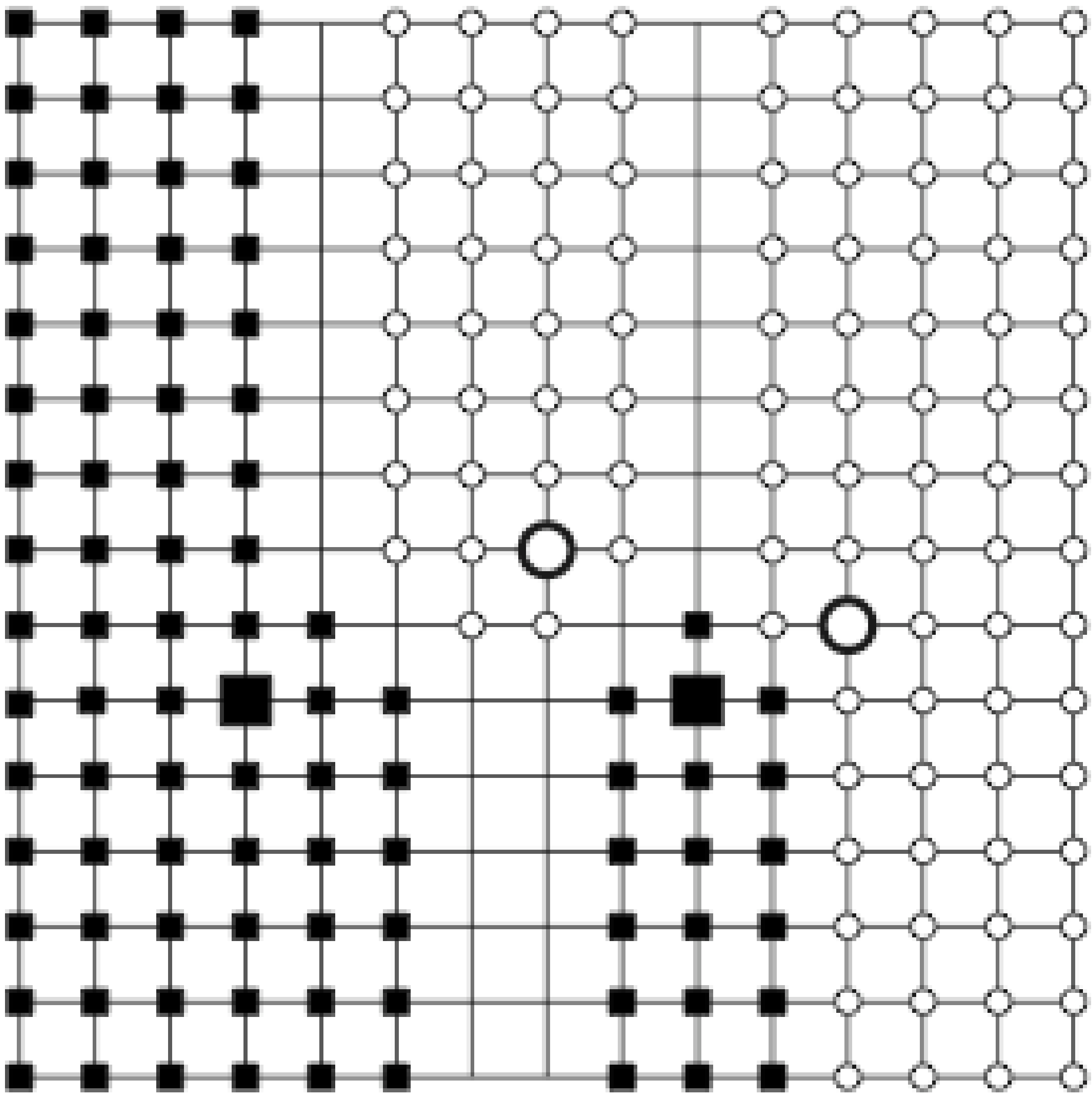
  [Cheong,Har-Peled,Linial,Matoušek'02]
  [Fekete,Meijer'03]



Past work on graphs

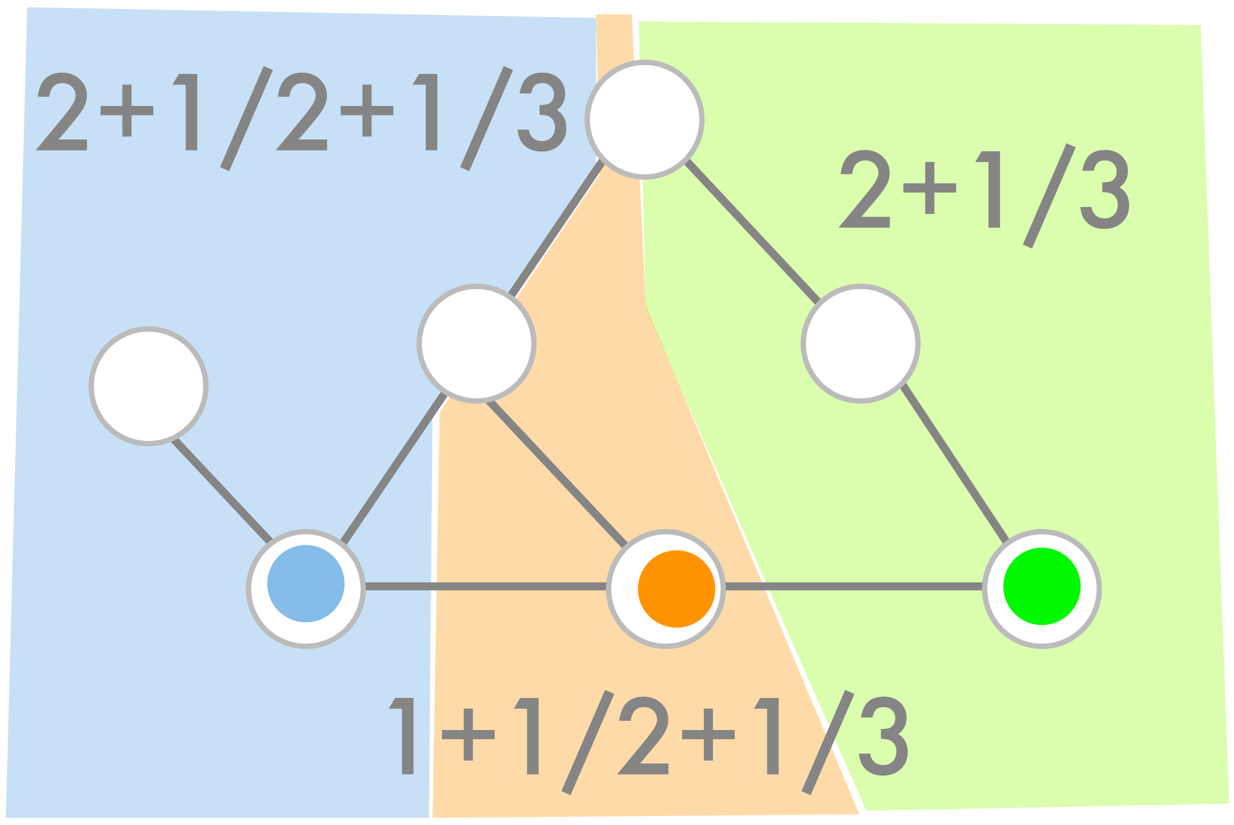
- For the same game on a graph, it is NP-complete to compute the best strategy for black (the 2nd player).

  [Teramoto,Demaine,Uehara'06]



What we studied

- discrete space : V with a distance given by G(V,E)
- k players, each is to place a single vendor on some vertex
- vertices are assigned to closest players, possible in equal fractions
- gain of a player = total amount of vertices assigned to to it
- strategy profile $\in V^k$, is a pure Nash equilibrium if no player can unilaterally increase its gain
- social cost = sum over all vertices of the distance to closest player = minimum k-median problem



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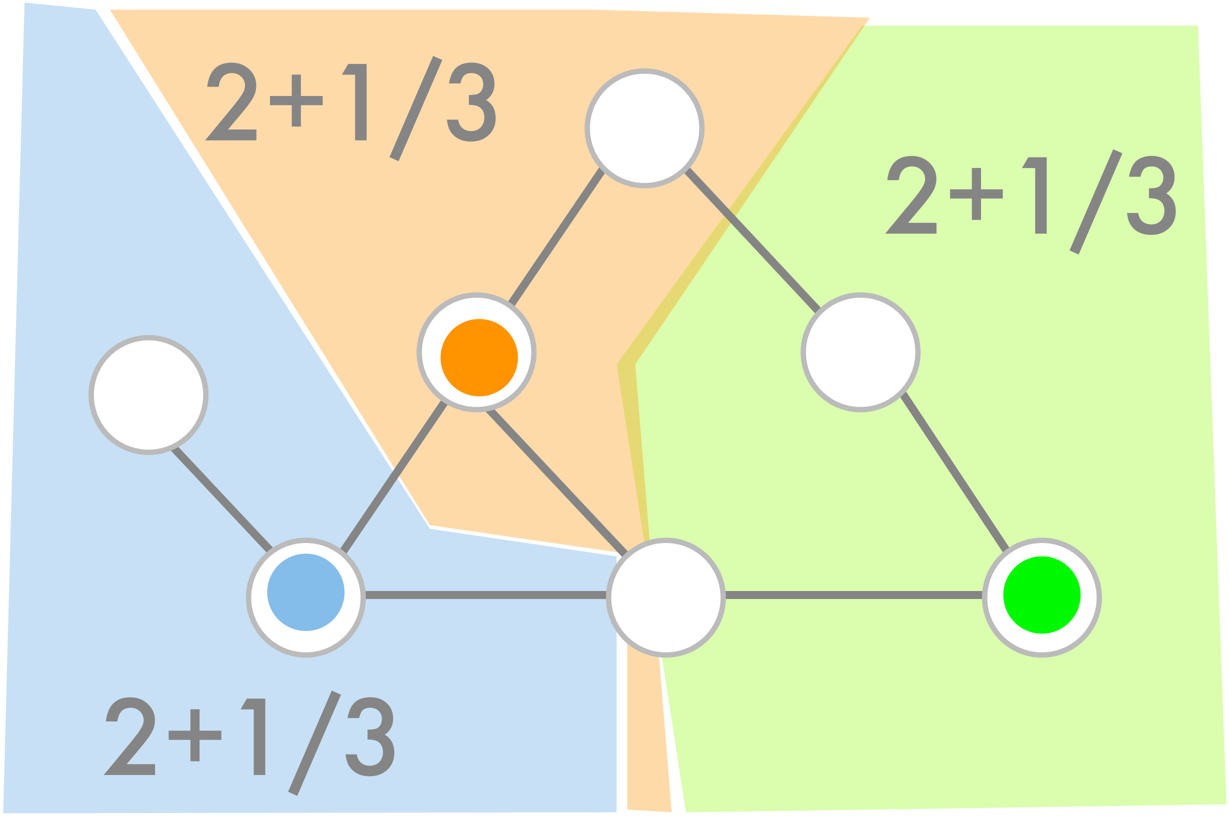
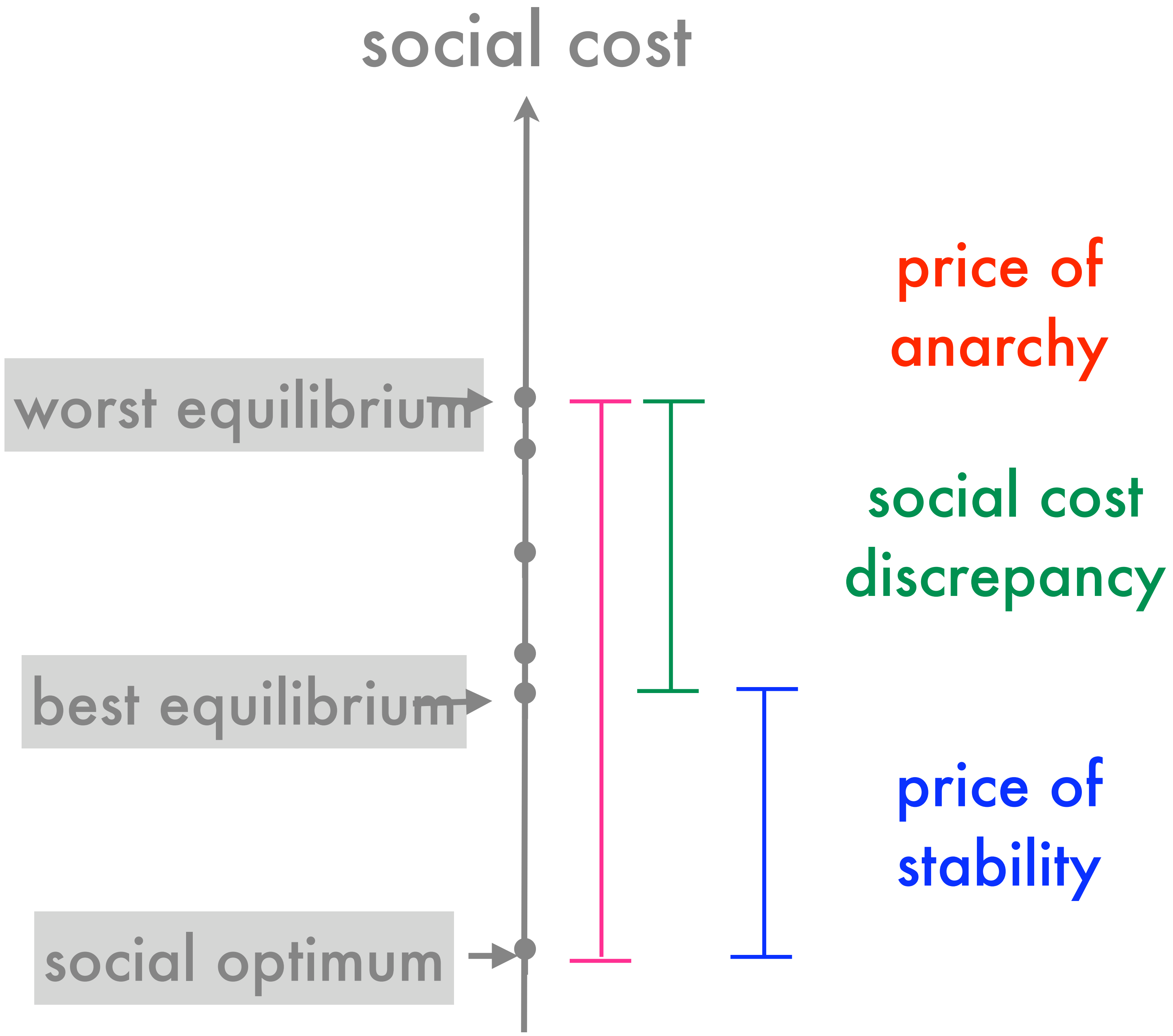


Price of anarchy



approximation ratio: price to pay for being restricted to polynomial time running time

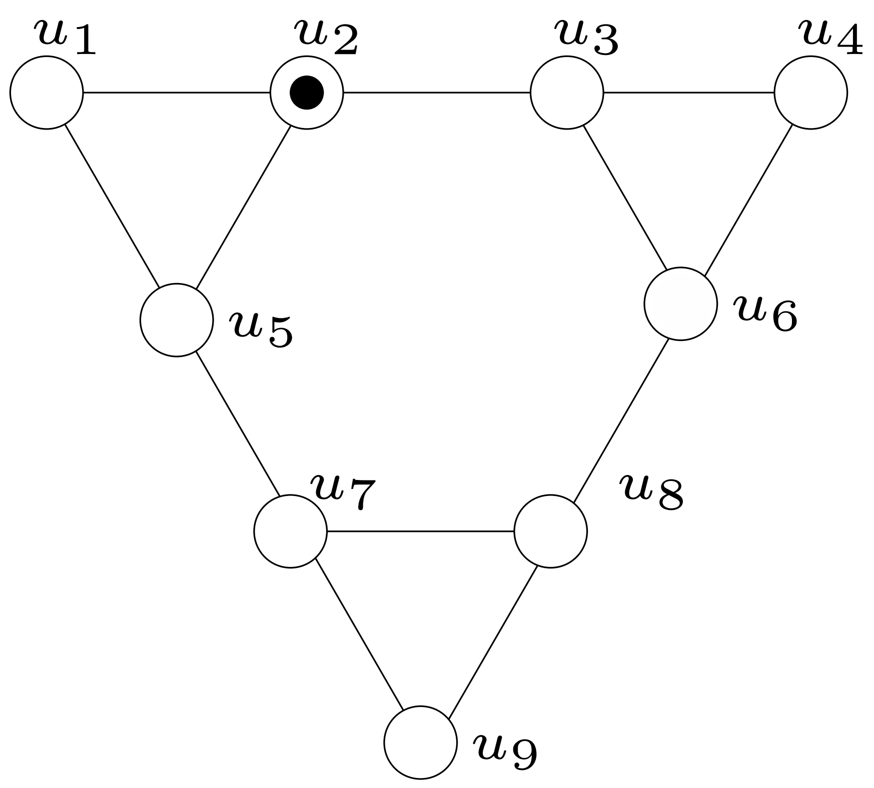
competitive ratio : price to pay for not knowing the future requests in advance

price of anarchy : price to pay for the lack of coordination

Does an equilibrium exist ?

- The existence of an equilibrium is a kind of graph property, it depends on *G(V,E)* but also on the number of players *k*. Our thm: deciding which is the case is NP-hard.

- For example this graph has no equilibrium for 2 players: Wlog, the 1st player places on u_1 or u_2. The the 2nd player can place on u6, and gain 5 (or 6), that's more than half. By sym. the 1st player can again change its strategy and gain more than half, and so on forever.



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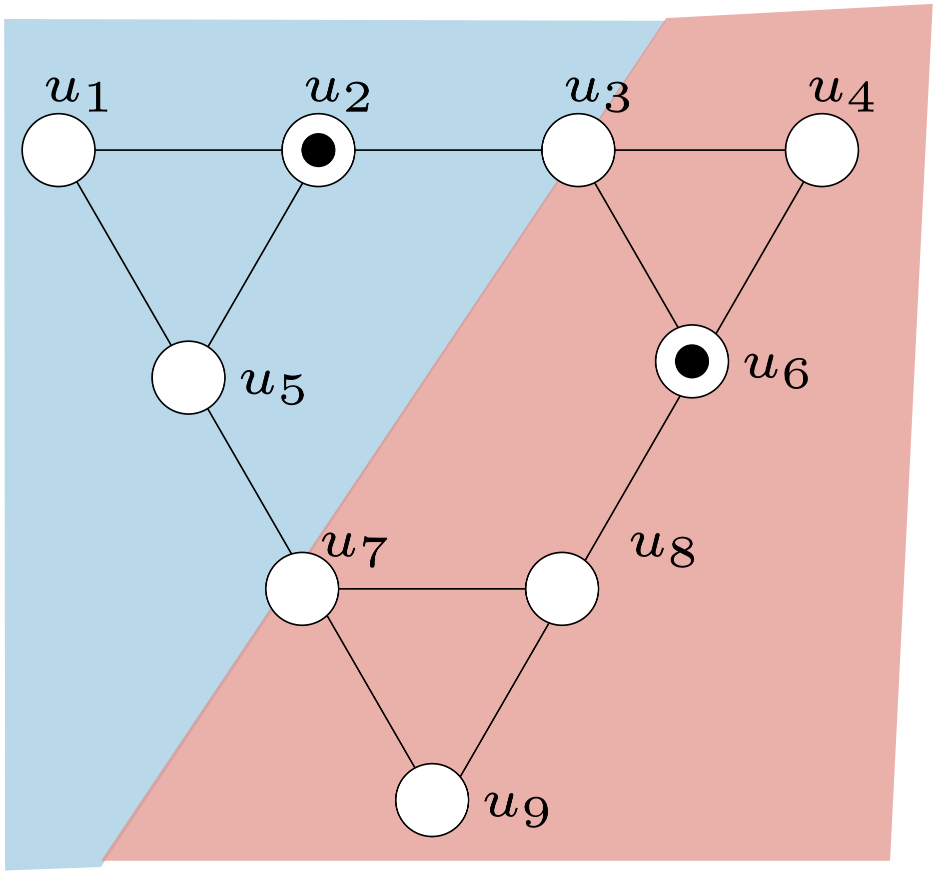
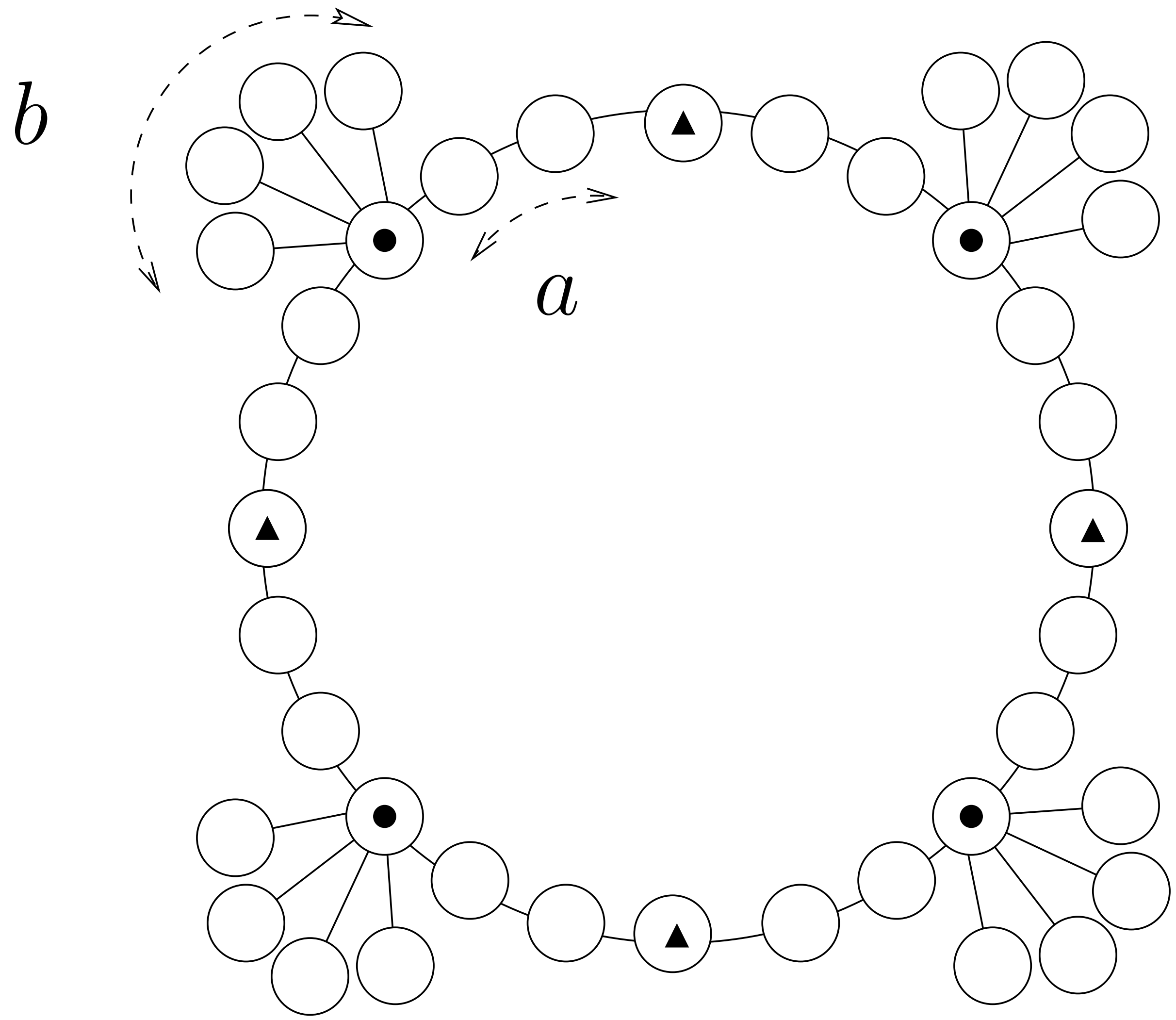


The social cost can differ by √(n/k)

How much do different equilibria differ with respect to social cost?



equilibrium ● *:*

cost $\Theta(kb+ka^2)$

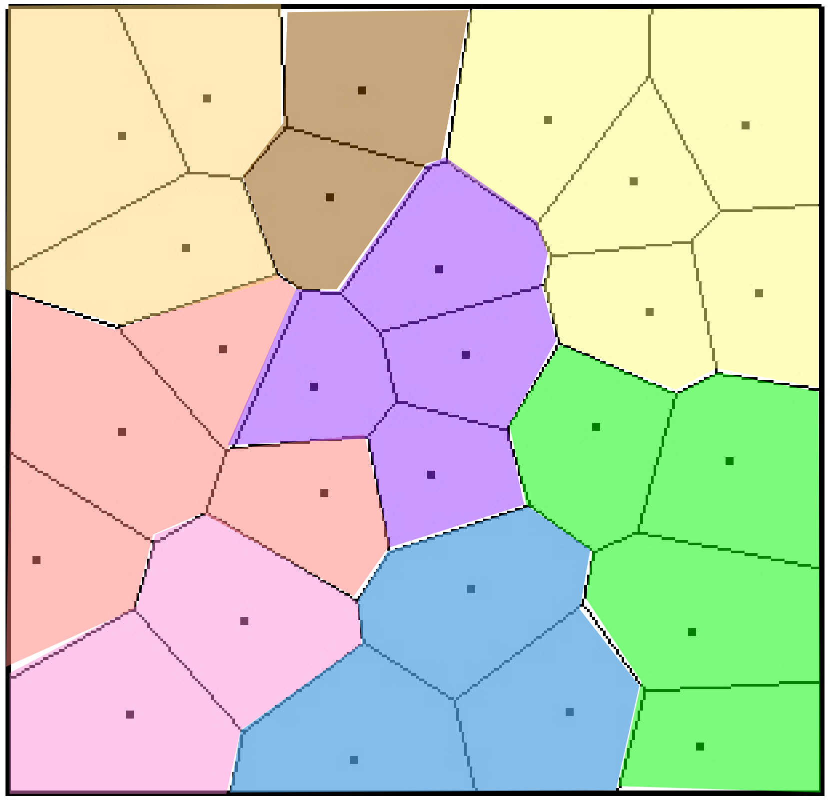
equilibrium ▲ *:*

cost $\Theta(kab+ka^2)$

worst ratio :

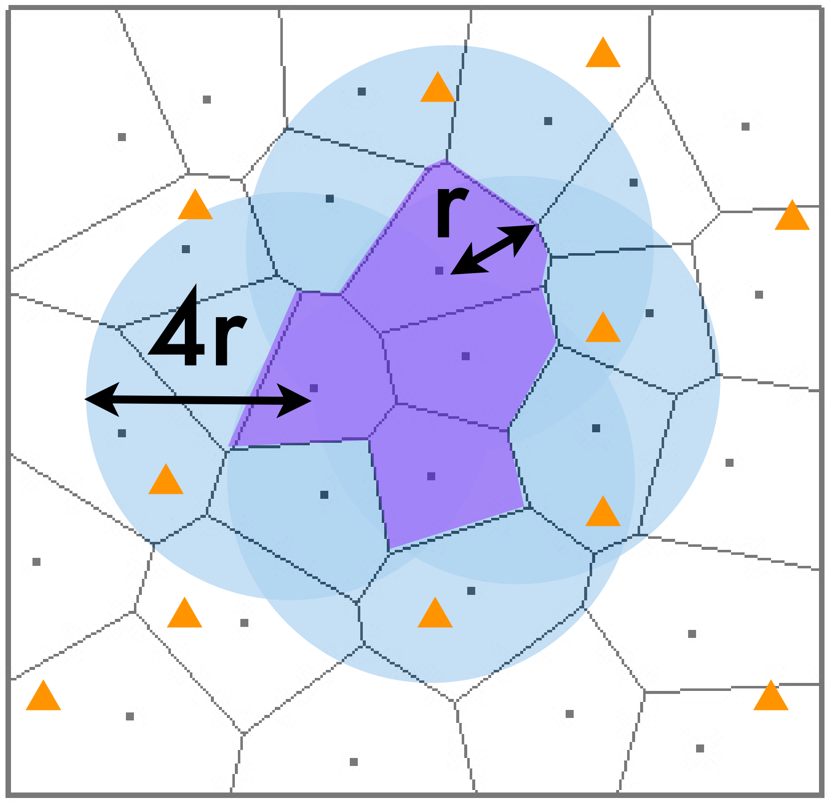
$\Omega(\sqrt{(n/k)})$ when $b=a^2$.

The social cost differs at most by √(kn)



- Let be 2 equilibria ● and ▲
- We group the Voronoi cells defined by ● into *regions*

The social cost differs at most by √(kn)



- For a fixed region, let r be the maximal distance from a vertex to the closest player
- We show that one of the ▲ players must be at distance at most 4r of every ● of this region

and now?

- Close the gap for the social cost discrepancy between $\sqrt{(n/k)}$ and $\sqrt{(kn)}$
- Find the price of anarchy of this game
- Understand the structure of the game already for simple graphs, trees or cycles.

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