Mecanism design for speed scaling scheduling

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Speed scaling scheduling

Given jobs:
- \( l: \) 
- \( j: \) priority \( p_j \)
- \( n: \) workload \( w_j \)

Single machine:

Decide on order on jobs and speed in order to minimize

energy consumption + weighted completion time

\[
\int s(t)^\alpha \, dt + \sum p_j C_j
\]

for a physical constant \( 2 \leq \alpha \leq 3 \)

Computational complexity is open but an approximation scheme exists [Megow, Verschae’13]
Define a strategic game

**deadline game**

- players decide on the deadline of their job (=strategies)

- compute minimum energy schedule = easy
- need to charge consumed energy to players

**penalty game**

- players announce a deadline penalty $\tilde{p}_i$ (=strategies)

- strategy proof is needed (dominant strategy should be $\tilde{p}_i = p_i$)
- compute minimum energy schedule = hard because we have to decide on the job order
- need to charge consumed energy to players
What do we want from a charging scheme?

1. compute optimal schedule (or approximate)

2. charge every user $i$ a value $b_i$

3. player $i$ wants to minimize $p_i C_i + b_i$

- pure Nash equilibria should exist
- … and be computable in polynomial time
- total amount charged should cover energy consumption and not exceed it by more than a constant factor (O(1)-budget balanced)
- social cost of equilibria should be close to social optimum (price of anarchy)
**Deadline Game**

### Proportional Charge

- Player $i$ pays exactly the energy consumed by his job
- Is clearly budget balanced
- Does not guarantee pure Nash equilibria

### Marginal Charge

- Player $i$ pays the difference of the optimal schedule with and without him
- Every player pays at least the energy consumed by his job and at most $\alpha$ times that value
- Is a potential game
  - Pure Nash equilibria exist, and can be found by best response dynamics,
  - Time of convergence has not been analyzed yet
- Price of anarchy has not been analyzed yet
deadline game
proportional cost sharing

• example with 2 identical jobs

• but any schedule creates an asymmetry between jobs

• every strategy profile \((d_1, d_2)\) is a point in \(\mathbb{R}^+ \times \mathbb{R}^+\)

• best response functions have no fix point

• there is no pure Nash equilibrium already for this simple game
deadline game, marginal cost share

- every player pays at least the energy consumed by his job and at most $\alpha$ times that value

- **tight example**: $n$ jobs with deadline 1, workload $1/n$.

- every player is charged $1 - (1 - 1/n)^\alpha$

- which is $\lim_{n \to \infty} 1 - (1 - 1/n)^\alpha = \alpha/n$
deadline game, marginal cost share

• \( \text{OPT}(d) \) = optimal energy consumption of a schedule for all players

• \( \text{OPT}(d_{-i}) \) = … all players but i

• cost share for player \( i \) = \( \text{OPT}(d) - \text{OPT}(d_{-i}) \)

• her total penalty is \( p_i d_i + \text{OPT}(d) - \text{OPT}(d_{-i}) \)

• but social cost is \( \Sigma p_i d_i + \text{OPT}(d) \)

• so if a player changes strategy and improves by \( \Delta \) so does the social cost

• this is a \textbf{potential game} \rightarrow \text{pure Nash equilibria exist}
penalty game

work in progress

• we need to fix an order on the jobs (arbitrary or random)
• then computing energy optimal schedule is easy
• cost share for player $i = \alpha (\text{OPT}(\tilde{p}) - \text{OPT}(\tilde{p}_{-i})) - \tilde{p}_i C_i$
• her total penalty is $(p_i - \tilde{p}_i) C_i + \alpha \text{OPT}(\tilde{p}) - \alpha \text{OPT}(\tilde{p}_{-i})$
• dominant strategy is $\tilde{p}_i = p_i$ (strategy proof)
• cost share is at least energy consumption of her jobs and at most $\alpha + 1$ times that value