WINE 2012

Smooth Inequalities and Equilibrium Inefficiency in Scheduling Games

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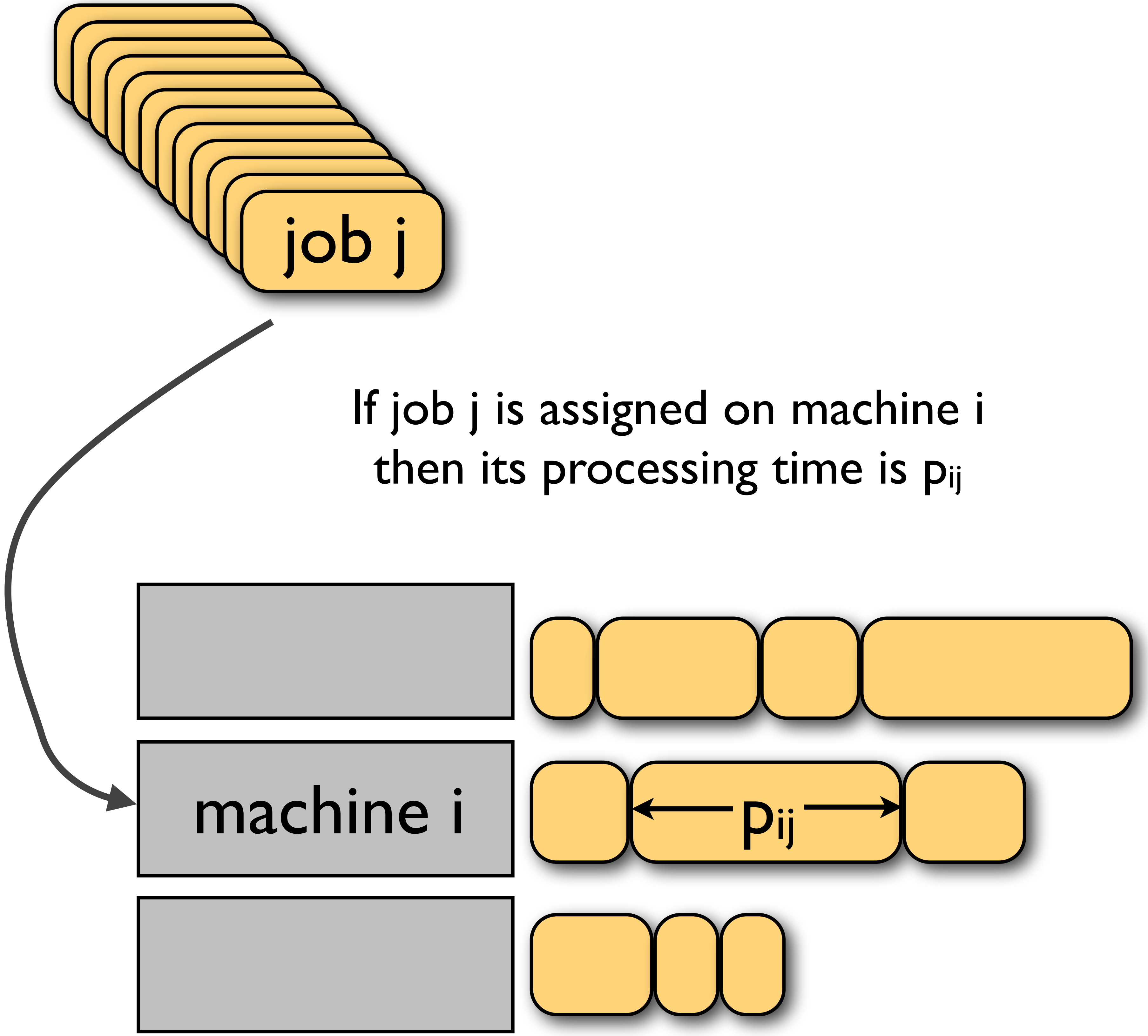
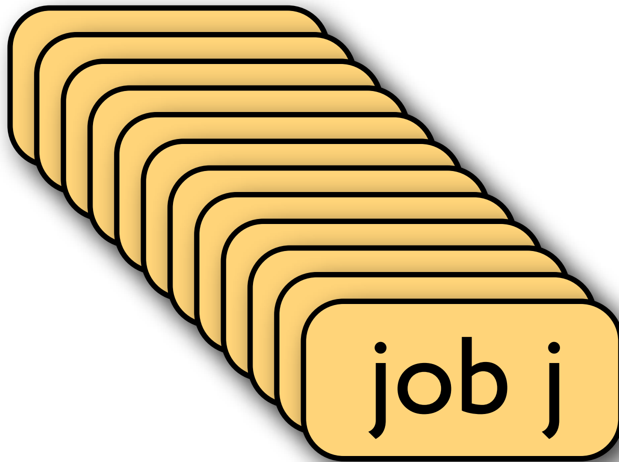
Scheduling: the unrelated machines model



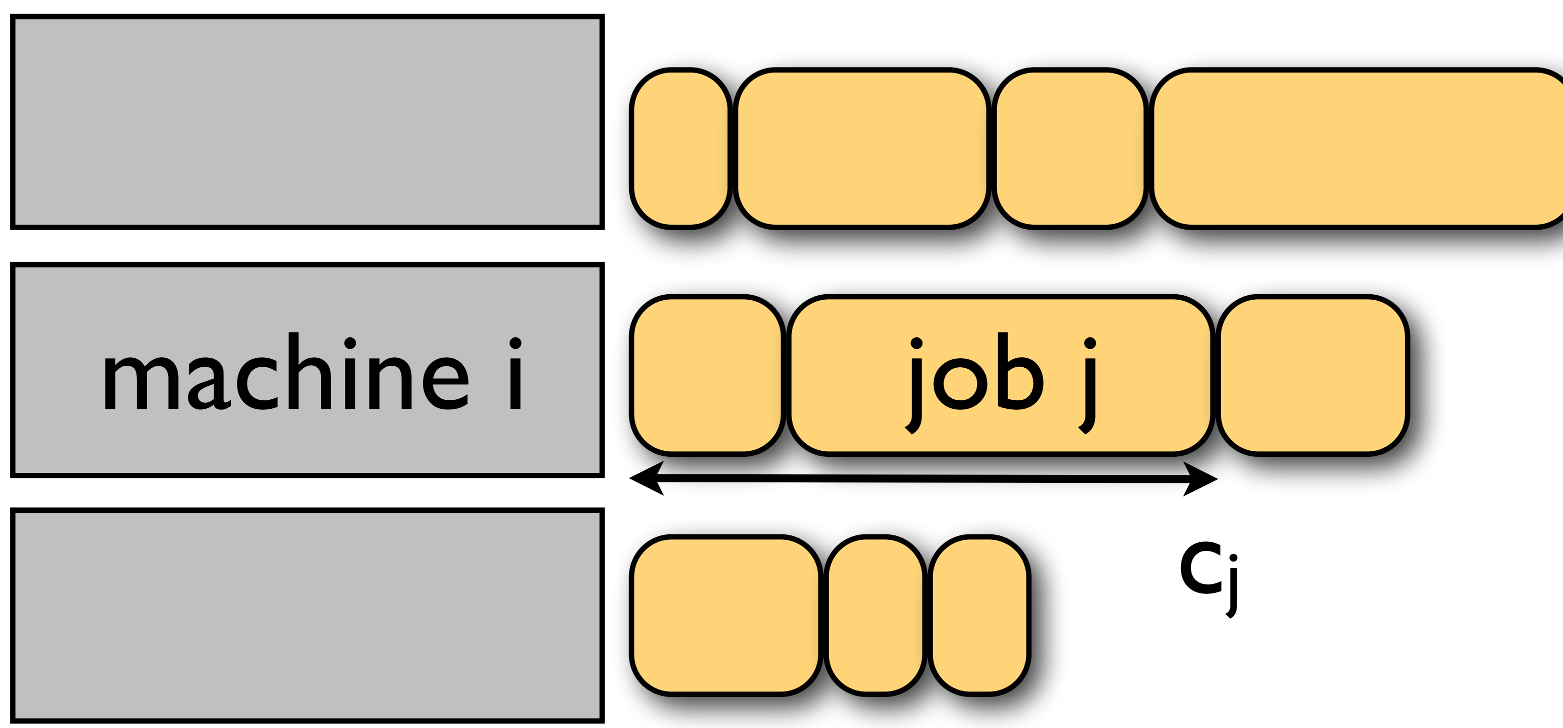
If job j is assigned on machine i then its processing time is p_{ij}

The game



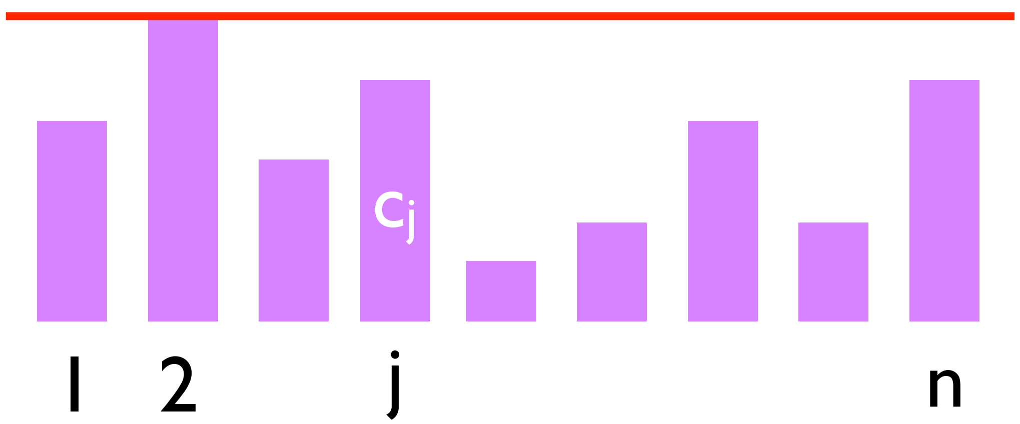
Jobs are players
they have to choose a machine
(strategy)
all they care is the completion
time c_j of their job j

As a game designer
we have to choose
and announce a
scheduling policy
that specifies how
jobs assigned to a
machine are to be
scheduled
(which order)

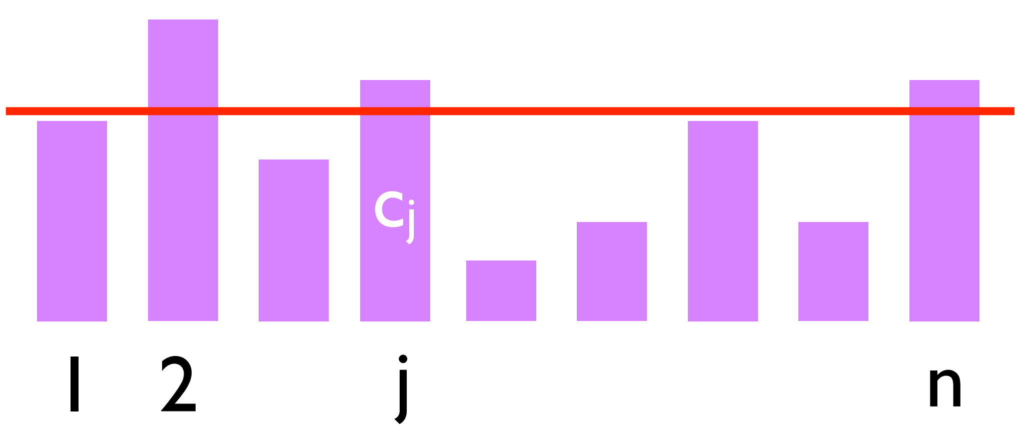


Social cost

What social cost do we want to optimize?

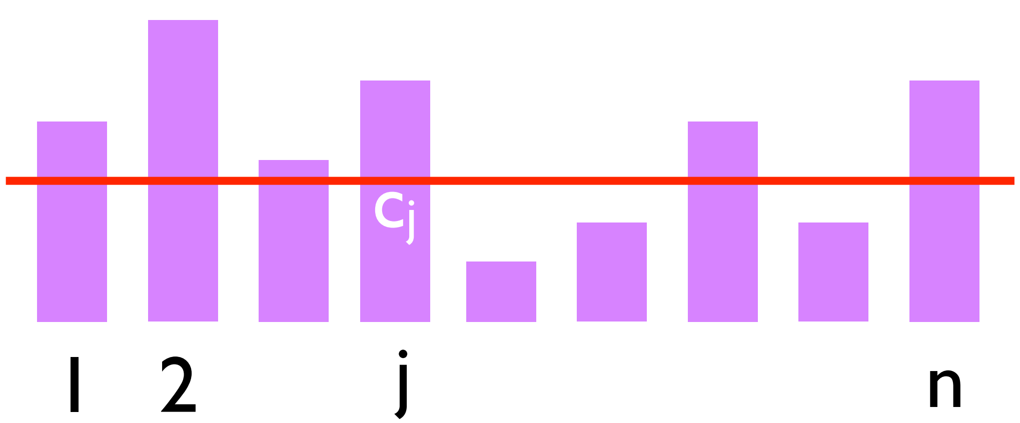


egalitarian objective:
max completion time, L_∞-norm



L_k-norm:

$$\sqrt[k]{\sum_j (c_j(x))^k}$$



utilitarian objective:
average completion time, L_1-norm

Smoothness argument

used by Christodoulou, Koutsoupias [STOC'2005] on congestion games
formalized by Tim Roughgarden [STOC'2009]

Game is (λ,μ)-smooth if for all strategy profiles x,x*

$$\sum_j c_j(x_j^*, x_{-j}) \leq \mu \sum_j c_j(x) + \lambda \sum_j c_j(x^*)$$

A (λ,μ)-smooth game with utilitarian social cost (L_1-norm) has price of anarchy at most

$$\frac{\lambda}{1-\mu}$$

for the proof use x arbitrary pure Nash equilibrium and x* the social optimum

$$C(x) \leq \sum_j c_j(x_j^*, x_{-j}) \leq \mu C(x) + \lambda C(x^*)$$

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$$\frac{\lambda}{1-\mu}$$

How to extend to the L_k-norm?
Hard to apply in the unrelated machine model.

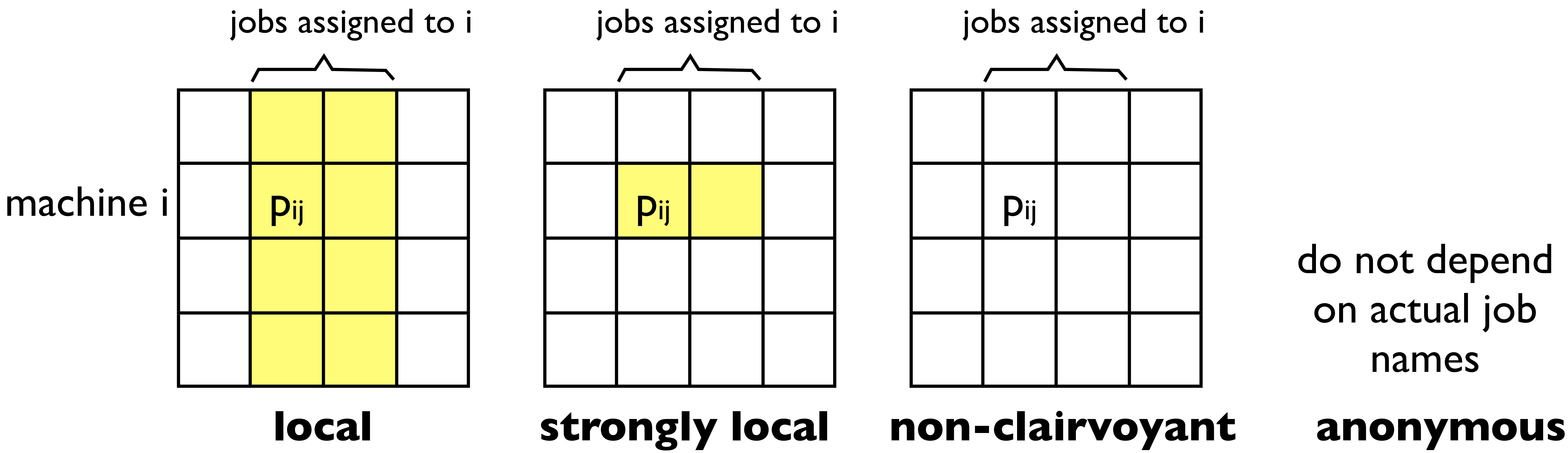
Our results

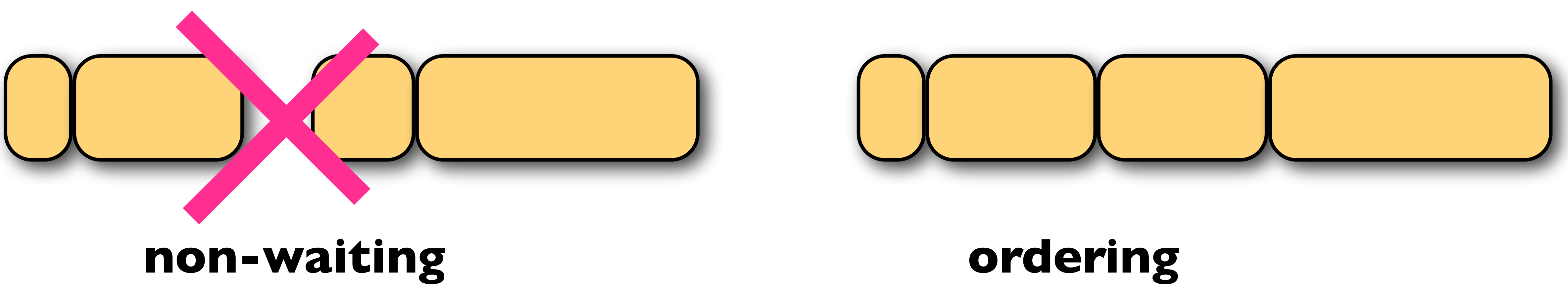
smooth inequalities

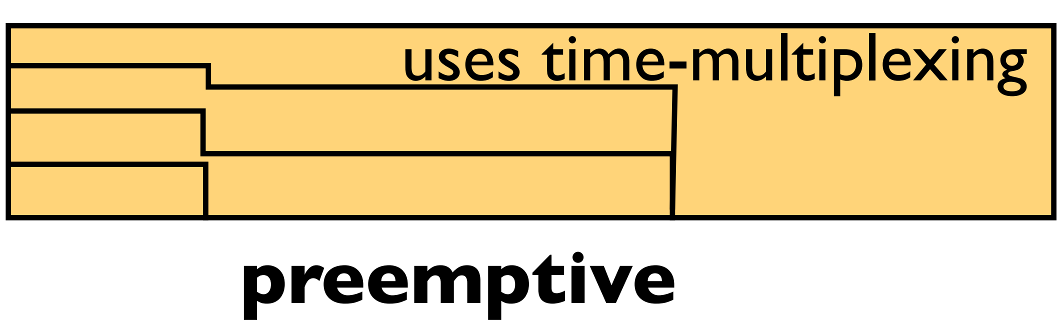
You don’t want to see them

A simple form was used by Suri, Tóth, Zhou,
“Selfish Load Balancing and Atomic Congestion Games”, 2007

Desired Properties of a Scheduling Policy

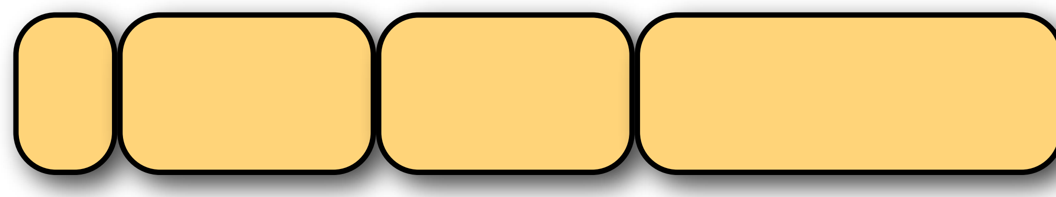






Scheduling Policies

SPT order from **s**hortest to longest **p**rocessing **t**ime



- many good properties:
- strongly local
- minimizes norm of completion times for every machine

Our results

for the L_k-norm social cost

SPT has price of anarchy $O(k^{\frac{k+1}{k}})$

any strongly local non-waiting policy has price of anarchy $\Omega(k^{\frac{k+1}{k}})$

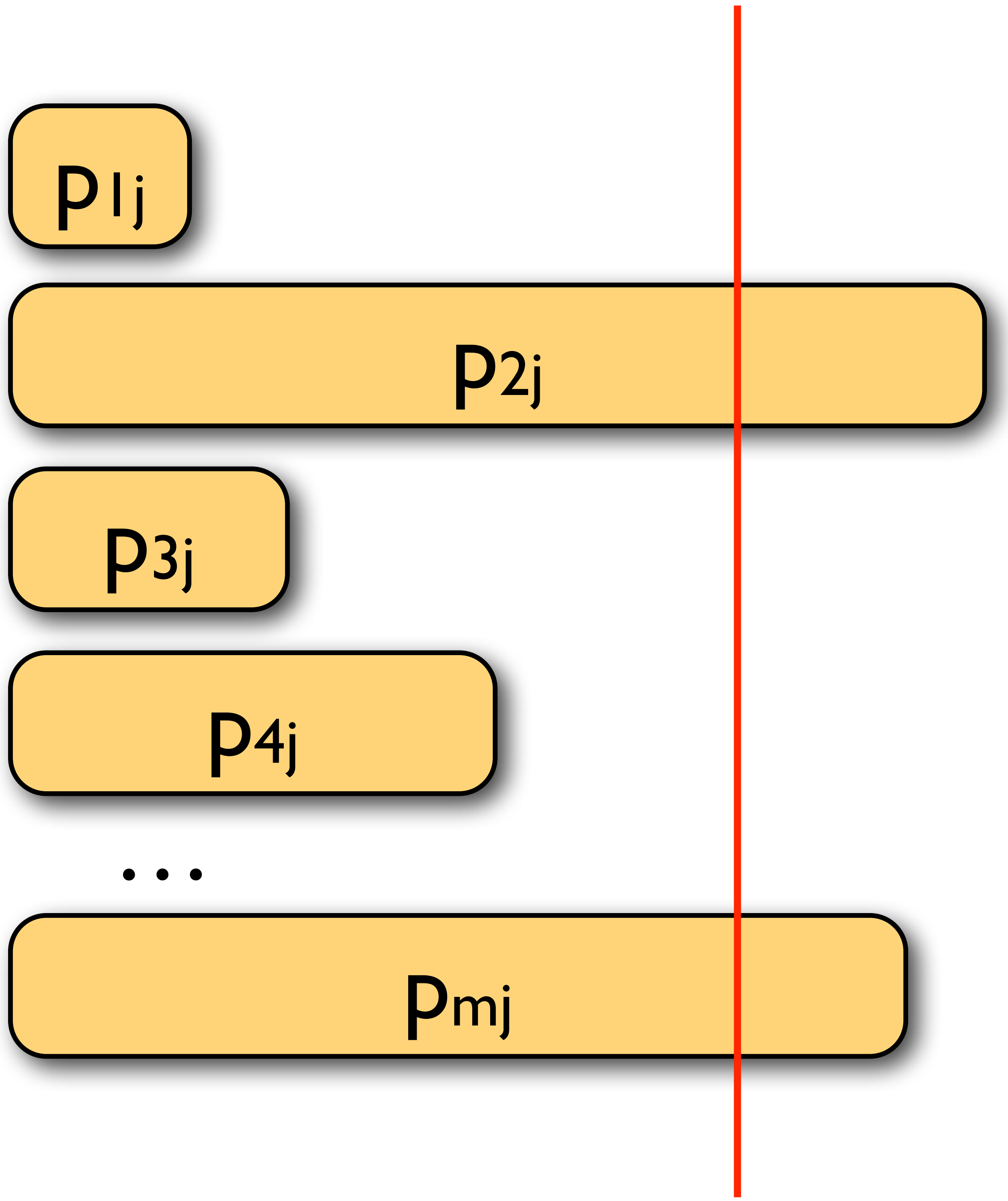
Our results

for the L_∞-norm social cost, and **m** unrelated machines

Azar,Jain,Mirrokni [SODA'2008]	$\Omega(m)$	ordering strongly local
Fleischer,Svitkina [2010]	$\Omega(\log m)$	ordering local
Abed,Huang [ESA'2012]	$\Omega(\log m/\log\log m)$	local
policy from Azar,Jain,Mirrokni [SODA'2008]	$O(\log^2 m)$	non-anonymous local
ACORD Caragiannis [SODA'2009]	$O(\log m)$	non-anonymous local
CCORD Caragiannis [SODA'2009]	$O(\log^2 m)$	anonymous local
BALANCE	$O(\log m)$	anonymous local

Definition of BALANCE

for the L_∞-norm social cost, and **m** unrelated machines



$q_j := \min\{p_{ij} : i = 1, \ldots, m\}$

$\rho_{ij} := p_{ij} / q_j$

We don't want a job j to choose a machine i where $\rho_{ij} > m$

Definition of BALANCE

for the L_∞-norm social cost, and **m** unrelated machines

$$c_j^h(x) = \begin{cases} \frac{1}{q_j}\left[\left(p_{ij} + \sum_{\substack{j':j' \prec_i j \\ x_{j'}=i}} p_{ij'}\right)^{h+1} - \left(\sum_{\substack{j':j' \prec_i j \\ x_{j'}=i}} p_{ij'}\right)^{h+1}\right] & \text{if } \rho_{ij} \leq m, \\ \infty & \text{otherwise.} \end{cases}$$

for $i=x_j$

$<_i$: is the ordering of jobs j in increasing p_{ij}, breaking ties according to local decision by machine i

This is a potential game.

For $h = \lfloor \log m \rfloor$ its price of anarchy is O(log m).

Thank you